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Global Exponential Stability and Stabilization of Fractional-Order Positive Switched Systems

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Abstract—This paper is concerned with the global exponential stability(GES) and stabilization of fractional-order positive switched systems(FOPSS) with average dwell time(ADT). Firstly, the the concept of GES is extended to FOPSS. Then, by constructing copositive Lyapunov functions and using ADT approach, a state feedback controller is constructed, and sufficient conditions are derived to guarantee that the corresponding closed-loop system is globally exponentially stable. Such conditions can be easily solved by linear programming. Finally, an example is given to demonstrate the effectiveness of the proposed method.

***Index Terms*—fractional-order positive switched systems, global exponential stability, average dwell time, linear programming.**

I. Introduction

Positive switched systems are a class of hybrid systems consisting of a family of positive subsystems and a switching law that specifies which subsystem will be activated along the system trajectory at each instant of time. Many remarkable results related to positive switched systems have been presented, see [1-6] and references therein. These results mentioned above refer to the positive switched systems with integer order derivative. In recent years, fracti- onal order systems have received much attention. This class of systems has been found many applications in the fields such as fractional-order biological system[7], fractional electrical networks[8-9], robotics[10], fractional- order Ch- uas circuit[11] and so on, fractional calculus is more feasible than integer calculations to model the behavior of such systems There have been many interesting results on fractional order systems. In [12,13], necessary and sufficient conditions for stability of fractional order systems were obt- ained by virtue of linear matrix inequalities. [14] considered the robust stability and stabilization of fractional-order linear systems with polytopic uncertainties. A method on finding an equivalent ordinary system of a fractional order system with order between 1 and 2 was proposed in [15].

Recently, some researchers have investigated the fractional-order positive systems[16-18] or fractional-order switched systems[19,20]. Only a few results about fractional-order positive switched systems have been presented[21-23]. [21] considered the controllability of FOPSS for fixed switching sequence. [22] considered the the problem of state-dependent switching control of FOPSS. [23] considered the guaranteed cost finite-time control of fractional-order positive switched systems. However, to the best of our knowledge, there is no result on the control problem of GES and stabilization for FOPSS with ADT, which motivates our study.

Motivated by the above discussions, in this paper, we are interested in investigating the problem of GES of FOPSS with ADT. Firstly, definition of GES is extended to FOPSS. Secondly, by using copositive type Lyapunov function and ADT approach, a static output feedback controller is designed. Thirdly, Some sufficient conditions are obtained to guarantee the corresponding closed-loop system is globally exponentially stable. Such conditions can be easily solved by linear programming. The paper is organized as follows. In Section 2, problem statements and necessary lemmas are given. UGES analysis and controller design are developed in Section 3. An numerical example is provided in Section 4. Finally, Section 5 concludes this paper.

Notations. Throughout this paper, $A \succ 0$ ($\succeq 0$, $\prec 0$, $\circ\ \mathbf{0}$) means that $a_{ij} > 0$ (≥ 0, < 0, ≤ 0), which is applicable to a vector. $A \succ B$ ($A \succeq B$) means that $A - B \succ 0$ ($A - B \succeq 0$); R is the n-dimensional vector space, the n-dimensional Euclidean space is denoted by R^n, $R^{n\times n}$ represents the space of $n \times n$ matrices with real entries. For a vector $x = (x_1, x_2, \ldots, x_n)^T$, its 1-norm is given by $\|x\|_1 = \sum_{k=1}^{n} |x_k|$, $|x| = (|x_1|, |x_2|, \ldots, |x_n|)^T$, where x_k is the k th element of the vector x. A^T is the transpose of matrix A. Matrices are

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assumed to have compatible dimensions for calculating if their dimensions are not explicitly stated.

II. PRELIMINARIES AND PROBLEM STATEMENTS

A. Fractional-order calculus

Fractional-order integro-differential operator is the generalization of integer order integro-differential operator. There are different definitions of the fractional-order integral or derivative. Given, $0<\alpha<1$, the uniform formula of a fractional integral is defined as

$$ {}_{t_0}D_t^{-\alpha} f(t)=\frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}\frac{f(\tau)}{(t-\tau)^{1-\alpha}}d\tau, \qquad (1)$$

where $\Gamma(\alpha)$ denotes the Gamma function with non-integer arguments. For $0<\alpha<1$, the Riemann-Liouville (RL) definition of fractional derivative is given as

$$ {}_{t_0}^{RL}D_t^{\alpha} f(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_{t_0}^{t}\frac{f(\tau)}{(t-\tau)^{\alpha}}d\tau, \qquad (2)$$

and Caputo definition of fractional derivative is given as

$$ {}_{t_0}^{C}D_t^{\alpha} f(t)=\frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{f'(\tau)}{(t-\tau)^{\alpha}}d\tau, \qquad (3)$$

where $f(t)$ is an arbitrary integrable function, ${}_{t_0}D_t^{-\alpha}$ is the fractional integral of order α on $[t_0,t]$, $\Gamma(\alpha)=\int_0^{\infty}e^{-t}t^{\alpha-1}dt$, ${}_{t_0}^{RL}D_t^{\alpha}$ and ${}_{t_0}^{C}D_t^{\alpha}$ represent Riema-nn-Liouville and Caputo fractional derivatives of order α of $f(t)$ on $[t_0,t]$, respectively. We mainly use these two fractional-order operators in this paper. From the above two definitions, we can obtain the following relation between them:

$$ {}_{t_0}^{RL}D_t^{\alpha} f(t)= {}_{t_0}^{C}D_t^{\alpha} f(t)+\frac{t^{-\alpha}}{\Gamma(1-\alpha)}f(t_0). \qquad (4)$$

Lemma 1 [23]. Let $\alpha\in(0,1)$, if $f(0)\ge 0$, then ${}_{t_0}^{RL}D_t^{\alpha} f(t)\le {}_{t_0}^{C}D_t^{\alpha} f(t)$.

B. Fractional-order positive switched systems

Consider the following FOPSS:

$$ {}_{t_0}^{C}D_t^{\alpha} x(t)=A_{\sigma(t)}x(t)+B_{\sigma(t)}u(t), \qquad (5)$$

where $x(t)\in R^n$ is the system state, $u(t)\in R^m$ represent the control input. ${}_{t_0}^{C}D_t^{\alpha}$ denotes Caputo fractional-order derivative. $\sigma(t):\ [0,\infty)\to \underline{S}=\{1,2,\ldots,S\}$ is the switching signal. S is the number of subsystems; $\forall p\in S$, A_p and B_p are constant matrices with appropriate dimensions, p denotes the pth systems and t_q denotes the qth switching instant.

Next, we will present some definitions, lemmas and inequalities for the FOPSS (5) for our further study.

Definition 1 [6]. The system (5) is said to be positive if for any switching signals $\sigma(t)$, any initial conditions $x(t_0)\succeq 0$, the corresponding trajectory satisfies $x(t)\succeq 0$ for all $t\succeq 0$.

Definition 2 [6]. A matrix A is called a Metzler matrix if the off-diagonal entries of matrix A are non-negative.

Lemma 2 [1]. A matrix is a Metzler matrix if and only if there exists a positive constant ς such that $A+\varsigma I_n\succeq 0$.

Definition 3 [23]. For any switching signals $\sigma(t)$ and any $T_2\ge T_1\ge 0$, let $N_{\sigma(t)}(T_1,T_2)$ denote the switching numbers of $\sigma(t)$ over the interval $[T_1,T_2)$. If there exist $N_0\ge 0$ and $T_\alpha>0$ such that

$$N_{\sigma(t)}(T_1,T_2)\le N_0+\frac{T_2-T_1}{T_\alpha} \qquad (6)$$

then T_α and N_0 are called ADT and chattering bound, respectively. Generally speaking, we choose $N_0=0$ in this paper.

Lemma 3 [22]. The system (5) is positive if and only if A_p, $\forall p\in \underline{S}$ are Metzler matrices and $\forall p\in \underline{S}$, $B_p\succeq 0$.

Definition 4 [1]. If there exist positive constants $\xi_1>0$ and $\xi_2>0$ such that the state response satisfies $\|x(t)\|_1\le \xi_1 e^{-\xi_2(t-t_0)}\|x(t_0)\|_1$, $\forall t\ge t_0$ with arbitrary nonnegative initial conditions, then system (5) is uniformly globally exponentially stable under a proper switching signal.

C. Some inequalities

The following inequalities are necessary for our further study.

Lemma 5. (Gronwall-bellman inequality) Let $a(t)$, $b(t)$ and $g(t)$ be continuous real-valued functions. If $a(t)$ is non-negative and if $g(t)$ satisfies the integral inequality

$$g(t)\le a(t)+\int_0^t b(s)g(s)ds,$$

then

$$g(t)\le a(t)+\int_0^t b(s)b(s)\exp(\int_s^t b(r)dr)ds.$$

If, in addition, $a(t)$ is a constant, then

$$g(t)\le a(t)+\exp(\int_0^t b(s)ds).$$

Lemma 6. (C_p inequality) For $0<\alpha<1$ and any positive real numbers x_1, x_2, …, x_k,

$$\sum_{k=1}^{n}x_k^{\alpha}\le n^{1-\alpha}(\sum_{k=1}^{n}x_k)^{\alpha}.$$

Lemma 7. (Young's inequality) If a and b are nonnegative real numbers and p and q are positive real numbers such that $1/p+1/q=1$, then

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$$ab \le \frac{a^p}{p} + \frac{b^q}{q}.$$

The aim of this paper is to design a state feedback controller $u(t) = K_{\sigma(t)}x(t)$ and a class of switching signals $\sigma(t)$ for FOPSS (5) such that the corresponding closed-loop system is uniformly globally exponentially stable.

III. MAIN RESULTS

A. Global exponential stability analysis

In this subsection, we will focus on the problem of GES for FOPSS (5) with $u(t) \equiv 0$.

Theorem 1. *Consider the system (5) with* $u(t) \equiv 0$. *Given positive constants* λ, μ, *if there exist positive vectors* v_p, $p \in \underline{S}$, *such that the following inequalities hold:*

$$A_p^T v_p \preceq -\lambda_p v_p \tag{7}$$

$$v_p \preceq \mu v_q \tag{8}$$

where $\forall p \in \underline{S}$, $v_p = [v_{p1}, v_{p2}, \dots, v_{pn}]$, $\mu \ge 1$, *the FOPSS (5) is globally exponentially stable with the ADT scheme*

$$T_\alpha > T_\alpha^* = \frac{\Gamma(\alpha+1)\ln\mu}{\lambda\alpha} \tag{9}$$

Proof. Constructing the multiple linear type Lyapunov-Krasovskii functional for the system (5) as follows:

$$V_{\sigma(t)}(t, x(t)) = x^T(t)v_{\sigma(t)} \tag{10}$$

where $v_p \in R_+^n$, $\forall p \in \underline{S}$.

Denote t_0, t_1, $\dots$ as the switching instants over the interval $[0, T_f]$. When $t \in [t_k, t_{k+1})$, along the trajectory of the system (5) with $u(t) = 0$, we have

$${}_{t_0}^{C}D_t^{\alpha} V_{\sigma(t)}(t, x(t)) = x^T(t)A_{\sigma(t)}^T v_{\sigma(t)} \tag{11}$$

From (7) and (11), we have

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} V_{\sigma(t)}(t, x(t)) &= x^T(t)A_{\sigma(t)}^T v_{\sigma(t)} \\ &\le -\lambda x^T(t)v_{\sigma(t)} \le -\lambda V_{\sigma(t)}(t, x(t)) \end{aligned} \tag{12}$$

Taking the fractional integral ${}_{t_0}^{C}D_t^{-\alpha}$ to both sides of (12) during the period $[t_k, t)$ for $t \in [t_k, t_{k+1})$ leads to

$$\begin{aligned} V_{\sigma(t)}(t, x(t)) &\le V_{\sigma(t_k)}(t_k, x(t_k)) \\ &\quad -\frac{\lambda}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}V_{\sigma(t)}(s, x(s))ds \end{aligned} \tag{13}$$

According to Lemma 5 and the properties of $\Gamma(\alpha)$, for $t \in [t_k, t_{k+1})$, we have

$$\begin{aligned} V_{\sigma(t)}(t, x(t)) &\le V_{\sigma(t_k)}(t_k, x(t_k)) \\ &\quad -\frac{\lambda}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}V_{\sigma(t)}(s, x(s))ds \\ &\le V_{\sigma(t_k)}(t_k, x(t_k))\exp\left\{\frac{-\lambda}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}ds\right\} \\ &= V_{\sigma(t_k)}(t_k, x(t_k))\exp\left\{\frac{-\lambda}{\alpha\Gamma(\alpha)}(t-t_k)^{\alpha}\right\} \\ &= V_{\sigma(t_k)}(t_k, x(t_k))\exp\left\{\frac{-\lambda}{\Gamma(\alpha+1)}(t-t_k)^{\alpha}\right\} \end{aligned} \tag{14}$$

For $t \in [t_k, t_{k+1}) V_{\sigma(t)} \le \mu V_{\sigma(t_k^-)}(t_k^-, x(t_k^-))$ is easily obtained from (8) and (10). From $\exp\left\{\frac{-\lambda}{\Gamma(\alpha+1)}(t-t_k)^{\alpha}\right\} > 0$, we have

$$\begin{aligned} &V_{\sigma(t)}(t, x(t)) \\ &\le \mu V_{\sigma(t_k^-)}(t_k^-, x(t_k^-))\exp\left\{\frac{-\lambda}{\Gamma(\alpha+1)}(t-t_k)^{\alpha}\right\} \end{aligned} \tag{15}$$

Then, for $t \in [t_0, t)$, from (14) and (15), we have

$$\begin{aligned} &V_{\sigma(t)}(t, x(t)) \\ &\le V_{\sigma(t_k)}(t_k, x(t_k))\exp\left\{\frac{-\lambda}{\Gamma(\alpha+1)}(t-t_k)^{\alpha}\right\} \\ &\le \mu V_{\sigma(t_k^-)}(t_k^-, x(t_k^-))\exp\left\{\frac{-\lambda}{\Gamma(\alpha+1)}(t-t_k)^{\alpha}\right\} \\ &\le \mu V_{\sigma(t_{k-1})}(t_{k-1}, x(t_{k-1}))\exp\left\{\frac{-\lambda}{\Gamma(\alpha+1)}[(t-t_k)^{\alpha} \right. \\ &\quad \left. +(t_k-t_{k-1})^{\alpha}]\right\} \\ &\le \mu^2 V_{\sigma(t_{k-1}^-)}(t_{k-1}^-, x(t_{k-1}^-))\exp\left\{\frac{-\lambda}{\Gamma(\alpha+1)}[(t-t_k)^{\alpha} \right. \\ &\quad \left. +(t_k-t_{k-1})^{\alpha}]\right\} \le \cdots \\ &\le \mu^{N_\sigma(t_0, t)} V_{\sigma(t_0)}(t_0, x(t_0))\exp\left\{\frac{-\lambda}{\Gamma(\alpha+1)}[(t-t_k)^{\alpha} \right. \\ &\quad \left. +(t_k-t_{k-1})^{\alpha} + \cdots + (t_1-t_0)^{\alpha}]\right\} \end{aligned} \tag{16}$$

By Lemma 6 and $\mu > 1$, we have

$$\begin{aligned} V_{\sigma(t)}(t, x(t)) &\le V_{\sigma(t_0)}(t_0, x(t_0))\exp\left\{\frac{t-t_0}{T_\alpha}\ln\mu \right. \\ &\quad \left. -\lambda\frac{(N_\sigma(t_0, t)+1)^{1-\alpha}(t-t_0)^{\alpha}}{\Gamma(\alpha+1)}\right\} \end{aligned} \tag{17}$$

According to Lemma 7, (17) can be rewritten as

$$V_{\sigma(t)}(t,x(t)) \le V_{\sigma(t_0)}(t_0,x(t_0))\exp\left\{\frac{t-t_0}{T_\alpha}\right.$$
$$\left.\cdot\ln\mu-\lambda\frac{(1-\alpha)(N_\sigma(t_0,t)+1)+\alpha(t-t_0)}{\Gamma(\alpha+1)}\right\}$$
$$=V_{\sigma(t_0)}(t_0,x(t_0))\exp\left\{\frac{t-t_0}{T_\alpha}\ln\mu\right. \tag{18}$$
$$\left.-\frac{\lambda(1-\alpha)}{\Gamma(\alpha+1)}\cdot\frac{(t-t_0)}{T_\alpha}-\lambda\frac{(1-\alpha)+\alpha(t-t_0)}{\Gamma(\alpha+1)}\right\}$$

From the Lyapunov functional (10), we have

$$\varepsilon_1\|x(t)\|_1 \le V_{\sigma(t)}(t,x(t)) \tag{19}$$

$$V_{\sigma(0)}(0,x(0)) \le \varepsilon_2\|x(0)\|_1 \tag{20}$$

From (18)-(20), we can get

$$\varepsilon_1\|x(t)\|_1 \le V_{\sigma(t)}(t,x(t))$$
$$\le \varepsilon_2\|x(0)\|_1\exp\left\{\left(\frac{\ln\mu}{T_\alpha}-\frac{\lambda\alpha}{\Gamma(\alpha+1)}\right)(t-t_0)\right\} \tag{21}$$

It follows that

$$\|x(t)\|_1$$
$$\le\frac{\varepsilon_2}{\varepsilon_1}\|x(0)\|_1\exp\left\{\left(\frac{\ln\mu}{T_\alpha}-\frac{\lambda\alpha}{\Gamma(\alpha+1)}\right)(t-t_0)\right\} \tag{22}$$
$$=\xi_1\exp\{-\xi_2(t-t_0)\}\|x(0)\|_1$$

where $\xi_1=\dfrac{\varepsilon_2}{\varepsilon_1}$ and $\xi_2=\dfrac{\lambda\alpha}{\Gamma(\alpha+1)}-\dfrac{\ln\mu}{T_\alpha}$.

According to Definition 4, the FOPSS system (5) is globally exponentially stable. This completes the proof.

B. State feedback controller design

In this section, we focus on the problem of controller design of the system (5). A state feedback controller will be designed to ensure the corresponding closed-loop system (23) is globally exponentially stable.

Consider the system (5), under the controller $u(t)=K_{\sigma(t)}x(t)$, the corresponding closed-loop system is given by

$${}_{t_0}^{C}D_t^\alpha x(t)=(A_{\sigma(t)}+B_{\sigma(t)}K_{\sigma(t)})x(t) \tag{23}$$

According to Lemma 2, to guarantee the positivity of the system (23), $A_p+B_pK_p$ should be Metzler matrices, $\forall p\in\underline{S}$. Theorem 2 gives some sufficient conditions to guarantee that the corresponding closed-loop system (23) is globally exponentially stable.

Theorem 2. *Consider the FOPSS (23). For given constants λ and μ, if there exist positive vectors v_p, $p\in\underline{S}$, such that (8) and the following conditions hold:*

$$A_p+B_pK_p \text{ are Metzler matrices} \tag{24}$$

$$A_p^Tv_p+f_p\preceq-\lambda_pv_p \tag{25}$$

where $\forall p\in\underline{S}$, $f_p=K_p^TB_p^Tv_p$, $v_p=[\,v_{p1},\ v_{p2},\ \dots,\ v_{pn}\,]$, $\mu\ge1$, then resulting closed-loop system (23) is globally exponentially stable with the ADT scheme (9).

Proof. By Lemma 2 and (24), we get the system (23) is positive. Replacing A_p in (7) with $A_p+B_pK_p$, letting $f_p=K_p^TB_p^Tv_p$, $\mu(\mu\ge1)$ satisfies (8), then under the ADT scheme (9), we easily obtain that the resulting closed-loop system (23) is globally exponentially stable.

The proof is completed.

Next, an algorithm is presented to obtain the feedback gain matrices K_p, $p\in\underline{S}$.

Step 1. By adjusting the parameters λ and solving (8) and (26) via linear programming, positive vectors v_p and f_p can be obtained.

Step 2. Substituting v_p and f_p into $f_p=K_p^TB_p^Tv_p$, K_p can be obtained.

Step 3. The gain K_p is substituted into $A_p+B_pK_p$. If $A_p+B_pK_p$ are Metzler matrices, then K_p are admissible. Otherwise, return to Step 1.

IV. NUMERICAL EXAMPLE

In this section, an example will be given to illustrate the effectiveness of the proposed method. Consider the system (5) with the parameters as follows:

$$A_1=\begin{bmatrix}-1.6&0\\0&-1.2\end{bmatrix},B_1=\begin{bmatrix}0.9&0\\0&1.0\end{bmatrix},$$
$$A_2=\begin{bmatrix}-1.5&0\\0&-1.3\end{bmatrix},B_2=\begin{bmatrix}1.2&0\\0&1.1\end{bmatrix},$$

Let $\alpha=0.9$, $\mu=1.4$, $\lambda=0.2$. Solving the inequalities in Theorem 2 by linear programming, we have

$$v_1=\begin{bmatrix}1.8764\\1.6843\end{bmatrix},v_2=\begin{bmatrix}1.6315\\1.7554\end{bmatrix},$$
$$f_1=\begin{bmatrix}1.3122\\1.2087\end{bmatrix},f_2=\begin{bmatrix}1.1518\\1.2356\end{bmatrix},$$
$$K_1=\begin{bmatrix}0.7340&0.7176\\0.7340&0.7176\end{bmatrix},K_2=\begin{bmatrix}0.5883&0.6399\\0.5883&0.6399\end{bmatrix},$$
$$A_1+B_1K_1=\begin{bmatrix}-0.9394&0.6458\\0.7340&-0.4824\end{bmatrix},$$
$$A_2+B_2K_2=\begin{bmatrix}-0.7940&0.7679\\0.6471&-0.5961\end{bmatrix}.$$

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It is easy to verify that $A_p + B_p K_p$ are Metzler matrices for each $p \in \underline{S}$. Then, according to (9), we can obtain $T_\alpha^* = 1.9976$. Choosing $T_\alpha = 2 > T_\alpha^*$. Under the state feedback controller, the simulation results are shown in Figs 1-3. The initial conditions of the system (5) are $x(0) = [0.5, 0.3]^T$. The state trajectories of the open-loop system are shown in Fig 1. According to the ADT scheme, the switching signals $\sigma(t)$ is depicted in Fig 2. The state trajectories of the closed-loop system with ADT are shown in Fig 3.

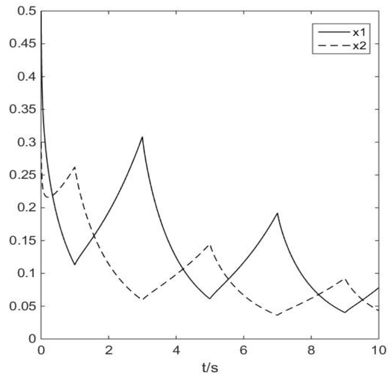

Fig 1: State trajectories of open-loop system (5).

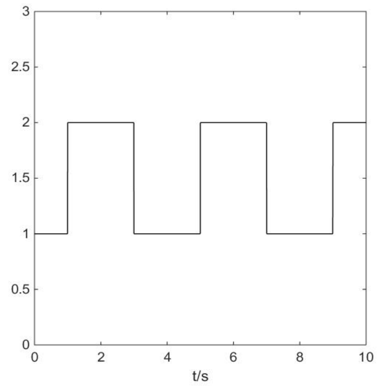

Fig 2: Switching signals of system (5) with ADT.

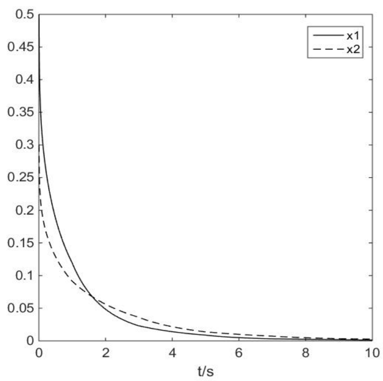

Fig 3: State trajectories of closed-loop system (5).

V. Conclusions

In this paper, we have studied the problem of GES and stabilization of FOPSS with ADT. By using ADT approach and constructing multiple linear copositive Lyapunov functions, a state feedback controller is designed, then a series of switching signals and some sufficient conditions are obtained to guarantee that the corresponding closed-loop system is globally exponentially stable. Such sufficient conditions can be solved by linear programming. Finally, an numerical example is provided to show the effectiveness of the proposed method.

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