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Epidemic drinking model: a dynamical analysis employing the Mittag-Leffler kernel

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Abstract: One of the most helpful operators for using fractional differential equations to explain non-local behaviors is the fractal fractional with Mittage-Leffler kernel fractional derivative. We proposed the fractal fractional Mittage-Leffler kernel for the drinking epidemic which is the world wide largest issue now a days. Qualitative and quantitative analysis for model and scheme are analyzed for drinking system in community. Additionally, using several well-known theorems from fixed point theory, we confirm the existence, uniqueness, analysis, and Ulam-Hyres stability of solutions to a specific class of fractional operators. Connect the drinking fractional system to a recently proposed bivariate Mittag-Leffler function to solve it. Some important properties were also verified for Mittage-Leffler fractional derivative on the fractional drinking system. Comparison has been made to the integer order derivative to verify the efficiency of results and effect of drinkers in community.

Keywords: Drinking Model; Fixed Point Theorems; Ulam-Hyres Stability; Mittag-Leffler kernel.

1. Introduction

Alcohol, often known as ethanol, has several detrimental effects on health. Short-term alcohol use might result in dehydration and intoxication. The long-term effects of alcohol use include altered liver and brain metabolism, several types of cancer, and alcohol use disorders. Alcohol intoxication affects the brain, leading to delayed reflexes, clumsiness, and slurred speech. Alcohol increases the production of insulin, which speeds up glucose metabolism and can cause low blood sugar, which can make diabetics irritable and even cause their death. Teenagers who still have growing brains are more susceptible to having an alcohol consumption disorder. Teenagers who drink are more likely to suffer harm, even death. A large amount of practical mathematics is devoted to the study of differential equations and their solutions. A differential equation, either ordinary or partial, can be used to model almost any dynamic process in nature. The monograph that Kilbas et al. [1] worked on covers the latest and most current research on fractional differential and fractional integro-differential equations using a variety of potentially helpful fractional calculus operators. The study of integrals and derivatives of any arbitrary real or complex order, as well as its applications, is known as fractional calculus. Sun et al. [2] continued to work in many scientific and technical fields where nonlocality is important in fractional calculus, there are still a lot of models that need to be developed, investigated, and used in real-world settings. Even while researchers have previously documented many astonishing results in important monographs and review papers, a large number of non-local phenomena remain unexplored and awaiting confirmation. Baleanu created a derivative with fractional order to identify the important issues while Ali Akgul worked on ABC technique. In this article, we present a novel approach to the analysis of fractional differential equations, which includes the fractional derivative of the Atangana-Baleanu issue [3]. EK Akgul's study's main goal is to solve linear and nonlinear fractional differential equations using the Mittag-Leffler nonsingular kernel. To solve this issue, a precise numerical approach has been developed. Two experiments are used to support the theoretical findings [4]. The derivatives are understood in the Caputo meaning by Ford et al. [5] the existence and uniqueness of solutions are discussed analytically first, and then we look at how the solutions relate to the available information. The unified transform technique, also called the Fokas approach, was developed by Fernandez et al. to solve partial differential equations. In order to apply the methodology to the solution of a sizable class of partial differential equations of fractional order, we adapt and modify it while incorporating new ideas as necessary. By using the approach to resolve a model fractional problem, we show how useful it is [6].

Alcohol ranks third internationally in terms of sickness and early mortality after low birth weight and hazardous sex. Dumitru Baleanu and Abdon Atangana were working on a novel fractional derivative with a non-local and non-singular kernel. We used the new derivative to solve the fractional heat transfer model and went over some of its beneficial features [7]. For the majority of enterprizes in Kenya, a Stephen et al. [8] considerable prevalence of alcohol issues in the workforce has been a growing source of concern. Due to issues related to alcoholism, the majority of employees exhibit erratic work attendance, low productivity, bad health, and safety hazards. A more realistic binge drinking model with time delay was developed by Huo HF et al. [9]. In our approach, the time lag of the immunity against drinking is represented by time delay. Routh-Hurwitz criteria for the model without a time delay. Despite significant advancements in oncological outcomes for patients with rectal cancer over the past few decades, M Grade et al. [10] found that high levels of impairment persisted in anorectal, urinary, and sexual functions regardless of whether radical surgery was carried out open or laparoscopically. Neurophysiological studies of 100 long-term alcoholics who were receiving neuropsychiatric care by Muller et al. [11] revealed signs of polytypic damage to the peripheral and central nervous systems. The findings demonstrate the need for a thorough diagnostic programme in order to identify the damage. Hasan et al. [12] investigated a complex area of mathematics called the fractional derivative for practical problems. The second coronavirus epidemic in India is the subject of this study. We create a time-fractional order COVID-19 model that includes a set of fractional differential equations to represent the effects of the disease. The Atangana Baleanu Caputo fractional derivative is used to study the fractional order COVID-19 model. The technique used by C Xu et al. [18] to uncover the changing law of bank data and efficiently manage banks is mathematical modeling. In this study, we developed a brand-new bank data model for fractional orders with two distinct time delays. Using the contraction mapping theorem, a mathematical analytical method, and the construction of a suitable function, we first discuss the existence and uniqueness, non-negativeness, and boundedness of the solution to the known bank data model. Research on the fractional-order model of HIV/AIDS is done by Farman et al. [19]. Caputo-Fabrizio and a fractional derivative operator with a compartment for antiviral treatment are used to examine this pandemic occurrence. For HIV/AIDS, the state-of-the-art approach is used. With the sumudu transform, a fractional-order model is able to produce results that are trustworthy. To validate the first responses, a qualitative examination of the fractional order HIV/AIDS model is carried out, and [20, 21] also examines various fractional applications using actual data.

The following are the remaining sections of this research paper: Section 1 contains the introduction and literature review. The basics of the hybrid fractional operator used in the model were discussed in Section 2. The suggested models with all the compartments are discussed in Section 3. Both the analysis of the suggested model in section 5 and the derivation and analysis of the drinking epidemic model utilizing the hybrid fractional operator are covered in Section 4. In section 6, we now go over the analysis of fractional integral operators of the suggested model. Lastly, succinct conclusions are provided in Section 7.

2. Basic Definition of Fractional Calculus

We obtained some important and useful conclusions in contemporary calculus and nonlinear dynamics [13].

Definition 2.1. *If $\Psi(t)$ is continuous over the range (a,b), then the fractal fraction integral of $\Psi(t)$ of order υ_1 with a kernel of type Mittag-Leffler is given by*

$$^{FFM}\mathbb{I}_{0,t}^{\upsilon_1,\upsilon_2}\psi(t) = \frac{\upsilon_2(1-\upsilon_1)t^{\upsilon_2-1}\psi(t)}{AB(\upsilon_1)} + \frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \rho^{\upsilon_2-1}\psi(\rho)(-\rho+t)^{\upsilon_1}d\rho. \tag{1}$$

Definition 2.2. *Given a Mittag-Leffler type kernel and the presumption that $\Psi(t)$ is continuous across the open interval (a,b), the fractal fraction derivative of $\Psi(t)$ of order υ_1 is given by*

$$^{FFM}\mathbb{D}_{0,t}^{\upsilon_1,\upsilon_2}\psi(t) = \frac{AB(\upsilon_1)}{1-\upsilon_1}\frac{d}{dt^{\upsilon_1}}\int_0^t \psi(\rho)\mathrm{E}_{\upsilon_1}\left\{\frac{\upsilon_1}{1-\upsilon_1}(-\rho+t)\right\}d\rho. \tag{2}$$

Lemma 2.1. *The following issue solution is defined for $\upsilon \in (0,1]$, per [21].*

$$\begin{cases} ^{ABC}\mathbb{D}_t^{\upsilon_1}\chi(t) = & \Omega(t), \\ \chi(0) = & \chi_0. \end{cases} \tag{3}$$

Therefore it is assumed that

$$\chi(t) = \chi_0 + \frac{1-\upsilon_1}{\eta(\upsilon_1)}\Omega(t) + \frac{\upsilon_1}{\Gamma(\upsilon_1)\eta(\upsilon_1)} \times \int_0^t (-\beta+t)^{\upsilon_1-1}\Omega(\beta)d\beta. \tag{4}$$

Theorem 2.1. *We next suppose that the DCP is a convex, bounded, and closed set and that P is a closed norm space. There is at least one fixed point in D for a continuous mapping $\Omega : D \to D$, and $\Omega D \subset P$ and ΩD are suitably compact.*

3. Mathematical Model

Significant ingredients of the model are detached here in the given points.

3.1. Model description

Our mathematical framework, which is shown in table 1, divides the population into four divisions.

Table 1: The classes in the suggested framework are described.

Compartments	**Definitions**
$\mathbf{S}(t)$	Non-Drinkers' Class
$\mathbf{H}(t)$	Category of Heavy Drinkers
$\mathbf{T}(t)$	Class of Drinkers in Treatment
$\mathbf{R}(t)$	Class of Recovered Drinkers

3.2. Model assumptions

The model was built under the following presumptions

1. The drinking pandemic takes place in a restricted setting.
2. The likelihood of becoming a heavy drinker is unaffected by a person's gender, race, or socioeconomic class.
3. When non-drinkers come into touch with heavy drinkers, heavy drinking is transferred to them.
4. Members who interact uniformly have a same degree of mixing.
5. Only once a patient has gone through the recovery and vulnerable compartments can they resume excessive drinking.
6. People who have given up drinking join the rehabilitation area.

Table 2 displays the full set of parameters for the proposed framework.

Table 2: The parameters of the suggested model are described.

Parameters	Definitions
b	**S** Rate of Recruitment
α	From **S** to **H** Transmission Rate
η	From **R** to **S** Transmission Rate
μ	Rate of Natural Death
δ_1	Drinking Increases the Risk of Death in **H**
δ_2	Drinking Increases the Risk of Death in **T**
ϕ	The percentage of drinkers who enter the **T** compartment
γ	Rate of Recovered **T**

4. Drinking Epidemic Model with Fractal Fractional Operator

We provide a deterministic compartmental model of the dynamics of transmission of drinking epidemics. Scientists are investigating the origin and recurrence of suggested epidemics. Let's examine some of the salient features of the compartmental mathematical epidemic model developed by C. Yang et al. [17] to explain viral transmission. The following collection of nonlinear ordinary differential equations represents the drinking epidemic model

$$\begin{cases} {}^{FFM}\mathbb{D}_t^{\upsilon}\mathbf{S}(t) = b - \alpha\mathbf{S}\mathbf{H} - \mu\mathbf{S} + \eta\mathbf{R}, \\ {}^{FFM}\mathbb{D}_t^{\upsilon}\mathbf{H}(t) = \alpha\mathbf{S}\mathbf{H} - (\mu + \delta_1 + \phi)\mathbf{H}, \\ {}^{FFM}\mathbb{D}_t^{\upsilon}\mathbf{T}(t) = \phi\mathbf{H} - (\mu + \delta_2 + \gamma)\mathbf{T}, \\ {}^{FFM}\mathbb{D}_t^{\upsilon}\mathbf{R}(t) = \gamma\mathbf{T} - (\mu + \eta)\mathbf{R}. \end{cases} \tag{5}$$

$$\mathbf{S}(0) = \mathbf{S}^0, \mathbf{H}(0) = \mathbf{H}^0, \mathbf{T}(0) = \mathbf{T}^0, \mathbf{R}(0) = \mathbf{R}^0. \tag{6}$$

Therefore, the entire population is determined by

$$\mathbb{N}(t) = \mathbf{H}(t) + \mathbf{S}(t) + \mathbf{R}(t) + \mathbf{T}(t). \tag{7}$$

4.1. Existence and Uniqueness Result

The fractal fractional derivative is written as $\upsilon_1, \upsilon_2 \in (0,1]$ and ${}^{FFM}\mathbb{D}_{0,t}^{\upsilon_1,\upsilon_2}$. Additionally, we derive the model's existence and uniqueness using the Atangana-Baleanu fractal fractional method. Our current goal is to establish that model (5) has at least one root. These representations for this considered model are feasible since the integral is differentiable:

$$\begin{cases} {}^{FFM}\mathbb{D}_{0,t}^{\upsilon_1,\upsilon_2}\mathbf{S}(t) = \mathrm{G}_1\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\}, \\ {}^{FFM}\mathbb{D}_{0,t}^{\upsilon_1,\upsilon_2}\mathbf{H}(t) = \mathrm{G}_2\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\}, \\ {}^{FFM}\mathbb{D}_{0,t}^{\upsilon_1,\upsilon_2}\mathbf{T}(t) = \mathrm{G}_3\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\}, \\ {}^{FFM}\mathbb{D}_{0,t}^{\upsilon_1,\upsilon_2}\mathbf{R}(t) = \mathrm{G}_4\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\}. \end{cases} \tag{8}$$

and also

$$\begin{cases} \mathrm{G}_1\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\} = b - \mathbf{S}\alpha\mathbf{H} - \mu\mathbf{S} + \mathbf{R}\eta, \\ \mathrm{G}_2\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\} = \mathbf{S}\alpha\mathbf{H} - (\delta_1 + \mu + \phi)\mathbf{H}, \\ \mathrm{G}_3\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\} = \phi\mathbf{H} - (\mu + \delta_2 + \gamma)\mathbf{T}, \\ \mathrm{G}_4\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\} = \gamma\mathbf{T} - (\mu + \eta)\mathbf{R}. \end{cases} \tag{9}$$

we can write equation (8) as:

$$\begin{cases} {}^{FFM}\mathbb{D}_{0,t}^{\upsilon_1,\upsilon_2}\Upsilon(t) = \upsilon_2 t^{\upsilon_2 - 1}\Phi(t, \Upsilon(t)), \\ \Upsilon(0) = \Upsilon_0. \end{cases} \tag{10}$$

Application of fractional integral results in:

$$\Upsilon(t)=\Upsilon(0)+\frac{\upsilon_2 t^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}+\frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \rho^{\upsilon_1-1}(1-\rho)^{\upsilon_2-1}\Phi(t,\Upsilon(t))d\rho, \tag{11}$$

where

$$\Upsilon(t)=\begin{cases}\mathbf{S}(t),\\ \mathbf{H}(t),\\ \mathbf{T}(t),\\ \mathbf{R}(t).\end{cases}\quad \Upsilon(0)=\begin{cases}\mathbf{S}(0),\\ \mathbf{H}(0),\\ \mathbf{T}(0),\\ \mathbf{R}(0).\end{cases}\quad \Phi(t,\Upsilon(t))=\begin{cases}\mathrm{G}_1\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\},\\ \mathrm{G}_2\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\},\\ \mathrm{G}_3\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\},\\ \mathrm{G}_4\{t,\mathbf{S},\mathbf{H},\mathbf{T},\mathbf{R}\}.\end{cases}$$

The definition of a Banach space in terms of the existence theory is $\mathbb{Y}\in[0,\mathbb{T}]$, where $\mathbb{W}=\mathbb{Y}\times\mathbb{Y}\times\mathbb{Y}\times\mathbb{Y}$. As a result, it adheres to the following standard:

$$\|\Upsilon(t)\|=\max_{t\in[0,\mathbb{T}]}|\mathbf{S}(t)+\mathbf{H}(t)+\mathbf{R}(t)+\mathbf{T}(t)|. \tag{12}$$

The operator $\xi:\mathbb{W}\to\mathbb{W}$ may be described as

$$\xi\Upsilon(t)=\Upsilon(0)+\frac{\upsilon_2 t^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}\Phi(t,\Upsilon(t))+\frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \rho^{\upsilon_1-1}(1-\rho)^{\upsilon_2-1}\Phi(t,\Upsilon(t))d\rho. \tag{13}$$

If $\Phi(t,\Upsilon(t))$ satisfies the Lipschitz condition and the ensuing extension,

- For every $\Upsilon\in\mathbb{W}$, there are constants K_p and J_p such that

$$|\Phi(t,\Upsilon(t))|\le K_p|\Upsilon(t)|+J_p \tag{14}$$

- For every $\Upsilon,\overline{\Upsilon}\in\mathbb{W}$, there exists a constant $M_p>0$ such that

$$|\Phi(t,\Upsilon(t))-\Phi(t,\overline{\Upsilon}(t))|\,|M_p|\le|\Upsilon(t)-\overline{\Upsilon}(t)| \tag{15}$$

Theorem 4.1. *Now the condition (14) is valid, then there exist at least single root to the proposed model (5) for the function $\Phi:[0,\mathbb{T}]\times\mathbb{Y}\to\mathbb{R}$.*

Proof. ξ is a continuous function since Φ in (13) is a continuous function.

We suppose that the expression $Z=\{\Upsilon\in\mathbb{Y}:\|\Upsilon\|\le\Re,\Re>0\}$, and we get $\Upsilon\in\mathbb{Y}$.

$$|\xi\Upsilon(t)|=\max_{t\in[0,\mathbb{T}]}\left|\Upsilon(0)+\frac{\upsilon_2 t^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}\Phi(t,\Upsilon(t))+\frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \rho^{\upsilon_1-1}(1-\rho)^{\upsilon_2-1}\Phi(t,\Upsilon(t))d\rho\right|.$$

$$\le\left|\Upsilon(0)+\frac{\upsilon_2\mathbb{T}^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}(K_p|\Upsilon(t)|+J_p)+\max_{t\in[0,\mathbb{T}]}\frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \rho^{\upsilon_1-1}(1-\rho)^{\upsilon_2-1}\Phi(t,\Upsilon(t))d\rho\right|.$$

$$\le\Upsilon(0)+\frac{\upsilon_2\mathbb{T}^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}(K_p|\Upsilon(t)|+J_p)+\frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}(K_p|\Upsilon(t)|+J_p)\mathbb{T}^{\upsilon_2+\upsilon_1-1}Z(\upsilon_2,\upsilon_1).$$

$$\le\Re.$$

Consequently, ξ is uniformly limited, and $\xi(\upsilon_1,\upsilon_1)$ is a β function. For equicontinuous of ξ, we select $t_1<t_2\le\mathbb{T}$, and then we assume

$$|\xi(\Upsilon)(t_2)-\xi(\Upsilon)(t_1)|=\left|\frac{\upsilon_2 t_2^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}\Phi(t_2,\Upsilon(t_2))+\frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^{t_2}\rho^{\upsilon_1-1}(1-\rho)^{\upsilon_2-1}\Phi(t,\Upsilon(t))d\rho\right.$$
$$\left.-\frac{\upsilon_2 t_1^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}\Phi(t_1,\Upsilon(t_1))-\frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^{t_1}\rho^{\upsilon_1-1}(1-\rho)^{\upsilon_2-1}\Phi(t,\Upsilon(t))d\rho\right|.$$

$$\leq \frac{\upsilon_2 t_2^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}(K_p\|\Upsilon(t_2)\|+J_p)+\frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\times(K_p\|\Upsilon(t_2)\|+J_p)t_2^{\upsilon_2+\upsilon_1-1}Z(\upsilon_1,\upsilon_2)$$
$$-\frac{\upsilon_2 t_1^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}(K_p\|\Upsilon(t_1)\|+J_p)-\frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\times(K_p\|\Upsilon(t_1)\|+J_p)t_1^{\upsilon_1+\upsilon_2-1}Z(\upsilon_1,\upsilon_1),$$

if $t_1 \to t_2$, then $|\xi\Upsilon(t_2)-\xi\Upsilon(t_1)| \to 0$. Consequently $\|\xi\Upsilon(t_2)-\xi\Upsilon(t_1)\| \to 0$ as $t_1 \to t_2$. Hence ξ is equicontinuous.

According to the Arzela Ascoli theorem, ξ is therefore fully continuous. There is at least one solution, as demonstrated by the conclusion of Schauder's fixed point. □

Theorem 4.2. *Suppose $\rho < 1$, where*

$$\rho = \left\{\frac{\upsilon_2 N^{\upsilon_2-1}(1-\upsilon_1)}{CD(\upsilon_1)}+\frac{\upsilon_1\upsilon_2}{CD(\upsilon_1)\Gamma(\upsilon_1)}M^{\upsilon_1+\upsilon_2-1}L(\upsilon_1,\upsilon_2)\right\}M_p.$$

The problem then has a unique solution for the investigated model.

Proof. For $\Upsilon, \overline{\Upsilon} \in \mathbb{W}$, we have

$$\left|\xi(\Upsilon)-\xi(\overline{\Upsilon})\right| = \max_{t\to[0,\mathbb{T}]}\left|\frac{\upsilon_2 t^{\upsilon_2-1}(1-\upsilon_1)}{CD(\upsilon_1)}\left(\Phi(t,\Upsilon(t))-\Phi(t,\overline{\Upsilon}(t))\right)+\frac{\upsilon_1\upsilon_2}{CD(\upsilon_1)\Gamma(\upsilon_1)}\right.$$
$$\left.\int_0^t \rho^{\upsilon_1-1}(1-\rho)^{\upsilon_2-1}\left[\Phi(t,\Upsilon(t))-\Phi(t,\overline{\Upsilon}(t))\right]d\rho\right|.$$
$$\leq \left[\frac{\upsilon_2 N^{\upsilon_2-1}(1-\upsilon_1)}{CD(\upsilon_1)}+\frac{\upsilon_1\upsilon_2}{CD(\upsilon_1)\Gamma(\upsilon_1)}N^{\upsilon_1+\upsilon_2-1}L(\upsilon_1,\upsilon_2)\right]\|\Upsilon-\overline{\Upsilon}\|.$$
$$\leq \rho\|\Upsilon-\overline{\Upsilon}\|.$$

Consequently, ξ is a contraction. For the proposed model, the Banach contraction principle offers an original solution. □

4.2. Analysis of Sensitivity

Given the information on the reproductive number $\mathbf{R}_0$ in [17], we obtain

$$\mathbf{R}_0 = \frac{a}{\mu+\delta_1+\phi}.$$

The sensitivity of $\mathbf{R}_0$ may be investigated by taking into account the partial derivatives of reproductive number for the pertinent parameters.

$$\frac{\partial \mathbf{R}_0}{\partial a} = \frac{1}{\mu+\delta_1+\phi},$$
$$\frac{\partial \mathbf{R}_0}{\partial \mu} = -\frac{a}{(\delta_1+\mu+\phi)^2},$$
$$\frac{\partial \mathbf{R}_0}{\partial \delta_1} = -\frac{a}{(\mu+\delta_1+\phi)^2},$$
$$\frac{\partial \mathbf{R}_0}{\partial \phi} = -\frac{a}{(\mu+\delta_1+\phi)^2}.$$

It is evident that R_0 is very responsive to parameter variations. In this study paper, a is increasing while μ, δ_1, and ϕ are dropping.

Sensitivity analysis may therefore lead us to the conclusion that prevention is the most effective way to manage illness.

4.3. Ulam-Hyres stability

In this portion of research article, we demonstrate the demonstration [22, 23]'s Ulam-Hyres stability.

Definition 4.1. *System (5) is Ulam Hyres stable if the positive operator* R, υ_1 *exists for any positive* ε *and for every* $\Upsilon \in (\mathbb{W}[0,T], \mathrm{R})$.

$$|{}^{FFM}\mathbb{D}_{0,t}^{\upsilon_1,\upsilon_2}\Upsilon(t) - \Phi(t,\Upsilon(t))| \leq \varepsilon, \qquad \forall t \in [0,T].$$

Thus, we get a special outcome. In this way, $\Upsilon \in (\mathrm{W}[0,T], \mathrm{R})$

$$|\Upsilon(t) - \Phi(t)| \leq \mathrm{R}_{\upsilon_1,\upsilon_2}\varepsilon, \qquad \forall t \in [0,T].$$

$\Phi(0) = 0$ if we assume a perturbation $\Psi \in \mathrm{W}[0,T]$. Now think about

- For $\varepsilon > 0$, we have $|\Phi(t)| \leq \varepsilon$.
- ${}^{FFM}\mathbb{D}_{0,t}^{\upsilon_1,\upsilon_2}\Upsilon(t) = \Phi(t,\Upsilon(t)) + \Psi(t)$.

Lemma 4.3. *A perturbed model has outcome*

$${}^{FFM}\mathbb{D}_{0,t}^{\upsilon_1,\upsilon_2}\Upsilon(t) = \Phi(t,\Upsilon(t)) + \Psi(t), \qquad \Psi(0) = \Psi_0.$$

has the connection

$$\left|\mathrm{R}(t) - \left\{\Upsilon(0) + \frac{\upsilon_2 t^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}\Phi(t,\Upsilon(t)) + \frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \rho^{\upsilon_2-1}(t-\rho)^{\upsilon_2-1}\Phi(\rho,\Phi(\rho))d\rho\right\}\right| \leq \chi_{\upsilon_1,\upsilon_2}\varepsilon.$$

$$\chi_{\upsilon_1,\upsilon_2}\varepsilon = \frac{\upsilon_2 T^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)} + \frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}T^{\upsilon_1+\upsilon_2-1}H(\upsilon_1,\upsilon_2).$$

Lemma 4.4. *If we take the Lipschitz condition of into account with lemma (4.3), under the condition* $\rho < 1$*, it has an UH stable solution.*

Proof. If $\upsilon_1 \in A$ represents a special outcome and $\Upsilon \in A$ represents any framework result,

$$|\Upsilon(t) - \upsilon_1(t)| = \left|\Upsilon(t) - \left\{\upsilon_1(0) + \frac{\upsilon_2 t^{\upsilon_1-1}(1-\upsilon_1)}{AB(\upsilon_1)}\Phi(t,\upsilon_1(t)) + \frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \rho^{\upsilon_2-1}(t-\rho)^{\upsilon_2-1}\Phi(\rho,\upsilon_1(\rho))d\rho\right\}\right|.$$

$$\leq \left|\Upsilon(t) - \left\{\upsilon_1(0) + \frac{\upsilon_2 t^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}\Phi(t,\Upsilon(t)) + \frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \rho^{\upsilon_2-1}(t-\rho)^{\upsilon_2-1}\Phi(\rho,\Upsilon(\rho))d\rho\right\}\right|.$$

$$\leq \left|\Upsilon(0) + \frac{\upsilon_2 t^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}\Phi(t,\Upsilon(t)) + \frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \rho^{\upsilon_2-1}(1-\rho)^{\upsilon_2-1}\Phi(\rho,\Upsilon(\rho))d\rho\right|$$
$$- \left|\upsilon_1(0) + \frac{\upsilon_2 t^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)}\Phi(t,\upsilon_1(t)) + \frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \rho^{\upsilon_2-1}(1-\rho)^{\upsilon_2-1}\Phi(\rho,\upsilon_1(\rho))d\rho\right|.$$
$$\leq \chi_{\upsilon_1,\upsilon_2}\varepsilon + \left\{\frac{\upsilon_2 T^{\upsilon_2-1}(1-\upsilon_1)}{AB(\upsilon_1)} + \frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}T^{\upsilon_1+\upsilon_2-1}H(\upsilon_1,\upsilon_2)\right\}L_\varepsilon\left|\Upsilon(t) - \upsilon_1(t)\right|.$$
$$\leq \chi_{\upsilon_1,\upsilon_2}\varepsilon + \rho|\Upsilon(t) - \upsilon_1(t)|.$$

Consequently,

$$\|\Upsilon - \upsilon_1\| \leq \chi_{\upsilon_1,\upsilon_2}\varepsilon + \rho\|\Upsilon(t) - \upsilon_1(t)\|.$$

Moreover, we can write the above expression as

$$\|\Upsilon - \upsilon_1\| \leq \mathrm{R}_{\upsilon_1,\upsilon_2}\varepsilon.$$

where $\mathrm{R}_{\upsilon_1,\upsilon_2} = \frac{\chi_{\upsilon_1,\upsilon\upsilon_2}}{1-\rho}$. It is Ulam-Hyres stable as a result. □

4.4. Analysis of equilibrium points

By setting the right side of the equation to zero and then solving for $\mathbf{S}^{\star}$, $\mathbf{H}^{\star}$, $\mathbf{T}^{\star}$ and $\mathbf{R}^{\star}$, we can assess the equilibrium points of the suggested model. We discovered in [17]:

$$\mathbf{S}^{\star} = \frac{\delta_1 + \mu + \phi}{\alpha}. \tag{16}$$

$$\mathbf{H}^{\star} = \frac{\mu(\mu + \delta_1 + \phi) - b\alpha}{\alpha[\eta - (\delta_1 + \mu + \phi)]}. \tag{17}$$

$$\mathbf{T}^{\star} = \frac{\phi[\mu(\delta_1 + \mu + \phi) - \alpha b]}{\alpha(\mu + \delta_2 + \gamma)[\eta - (\mu + \delta_1 + \phi)]}. \tag{18}$$

$$\mathbf{R}^{\star} = \frac{\gamma\phi[\mu(\delta_1 + \mu + \phi) - \alpha b]}{\alpha(\mu + \eta)(\mu + \delta_2 + \gamma)[\eta - (\mu + \delta_1 + \phi)]}. \tag{19}$$

5. Numerical Scheme with Mittage-Leffler Kernel

To illustrate the fractal fractional model (5) numerical algorithm [14, 15]. The following Cauchy problem was used to first define the overall system and then to provide the method

$$\begin{cases} {}^{FFM}\mathbb{D}_{0,t}^{\upsilon_1,\upsilon_2}\psi(t) = \Phi(t, \psi(t)), \\ \psi(0) = \psi_0. \end{cases} \tag{20}$$

By integrating the above equation, the formula shown below is generated.

$$\psi(t) = \psi(0) + \frac{(1-\upsilon_1)\upsilon_2 t^{\upsilon_2 - 1}\Phi(t, \psi(t))}{AB(\upsilon_1)} + \frac{\upsilon_1\upsilon_2}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \rho^{\upsilon_2-1}\Phi(\rho, \psi(\rho))(t-\rho)^{\upsilon_1-1}d\rho. \tag{21}$$

Now we consider $\mu(t, \psi(t)) = \upsilon_2 t^{\upsilon_2-1}\Phi(t, \psi(t))$, then system (13) becomes

$$\psi(t) = \psi(0) + \frac{(1-\upsilon_1)\mu(t, \psi(t))}{AB(\upsilon_1)} + \frac{\upsilon_1}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^t \mu(\rho, \psi(\rho))(t-\rho)^{\upsilon_1-1}d\rho. \tag{22}$$

The current value of $t_{n+1} = (n+1)\Delta t$ yields the following

$$\psi(t_{n+1}) = \psi(0) + \frac{(1-\upsilon_1)\mu(t_n, \psi(t_n))}{AB(\upsilon_1)} + \frac{\upsilon_1}{AB(\upsilon_1)\Gamma(\upsilon_1)}\int_0^{t_{n+1}} \mu(\rho, \psi(\rho))(t_{n+1}-\rho)^{\upsilon_1-1}d\rho. \tag{23}$$

also, we have

$$\psi(t_{n+1}) = \psi(0) + \frac{(1-\upsilon_1)\mu(t_n, \psi(t_n))}{AB(\upsilon_1)} + \frac{\upsilon_1}{AB(\upsilon_1)\Gamma(\upsilon_1)}\sum_{\iota=2}^{n}\int_{t_\iota}^{t_{\iota+1}} \mu(\rho, \psi(\rho))(t_{n+1}-\rho)^{\upsilon_1-1}d\rho. \tag{24}$$

By using the Newton polynomial to approximate the function $\mu(t, \psi(t))$, we have

$$\begin{aligned} Q_n(\rho) =& \mu(t_{-2+n}, \psi(t_{-2+n})) + \frac{\mu(t_{-1+n}, \psi(t_{-1+n})) - \mu(t_{-2+n}, \psi(t_{-2+n}))}{\Delta t}(\rho - t_{-2+n}) \\ &+ \frac{\mu(t_n, \psi(t_n)) - 2\mu(t_{-1+n}, \psi(t_{-1+n})) + \mu(t_{-2+n}, \psi(t_{-2+n}))}{2(\Delta t)^2}(\rho - t_{-2+n})(\rho - t_{n-1}). \end{aligned} \tag{25}$$

using equation (25) into system (24), we have

$$\begin{aligned} \psi^{n+1}(t) =& \psi_0 + \frac{(1-\upsilon_1)\mu(t_n, \psi(t_n))}{AB(\upsilon_1)} + \frac{\upsilon_1}{AB(\upsilon_1)\Gamma(\upsilon_1)}\sum_{\iota=2}^{n}\int_{t_\iota}^{t_{\iota+1}}\Big\{\mu(t_{-2+n}, \psi(t_{-2+n})) \\ &+ \frac{\mu(t_{-1+n}, \psi(t_{-1+n})) - \mu(t_{-2+n}, \psi(t_{-2+n}))}{\Delta t}(\rho - t_{-2+n}) \\ &+ \frac{\mu(t_n, \psi(t_n)) - 2\mu(t_{-1+n}, \psi(t_{-1+n})) + \mu(t_{-2+n}, \psi(t_{-2+n}))}{2(\Delta t)^2}(\rho - t_{-2+n})(\rho - t_{-1+n})\Big\}(t_{1+n} - \rho)^{\upsilon_1-1}d\rho. \end{aligned} \tag{26}$$

Rearranging the above system, we have

$$\psi^{n+1}(t) = \psi_0 + \frac{(1-\upsilon_1)\mu(t_n,\psi(t_n))}{AB(\upsilon_1)} + \frac{\upsilon_1}{AB(\upsilon_1)\Gamma(\upsilon_1)}\sum_{\iota=2}^{n}\Big\{\int_{t_\iota}^{t_{\iota+1}}\mu(t_{\iota-2},\psi^{\iota-2})(t_{1+\iota}-\rho)^{\upsilon_1-1}d\rho$$
$$+\int_{t_\iota}^{t_{\iota+1}}\frac{\mu(t_{\iota-1},\psi^{\iota-1})-\mu(t_{\iota-2},\psi^{\iota-2})}{\Delta t}(\rho-t_{\iota-2})(t_{\iota+1}-\rho)^{\upsilon_1-1}d\rho \tag{27}$$
$$+\int_{t_\iota}^{t_{\iota+1}}\frac{\mu(t_\iota,\psi^\iota)-2\mu(t_{\iota-1},\psi^{\iota-1})+\mu(t_{\iota-2},\psi^{\iota-2})}{2(\Delta t)^2}(\rho-t_{\iota-2})(\rho-t_{\iota-1})(t_{\iota+1}-\rho)^{\upsilon_1-1}d\rho\Big\}.$$

Writing further system (27), we have

$$\psi^{n+1}(t) = \psi_0 + \frac{(1-\upsilon_1)\mu(t_n,\psi(t_n))}{AB(\upsilon_1)} + \frac{\upsilon_1}{AB(\upsilon_1)\Gamma(\upsilon_1)}\sum_{\iota=2}^{n}\mu(t_{\iota-2},\psi^{\iota-2})\int_{t_\iota}^{t_{\iota+1}}(t_{\iota+1}-\rho)^{\upsilon_1-1}d\rho$$
$$+\sum_{\iota=2}^{n}\frac{\mu(t_{\iota-1},\psi^{\iota-1})-\mu(t_{\iota-2},\psi^{\iota-2})}{\Delta t}\int_{t_\iota}^{t_{\iota+1}}(\rho-t_{\iota-2})(t_{\iota+1}-\rho)^{\upsilon_1-1}d\rho \tag{28}$$
$$+\sum_{\iota=2}^{n}\frac{\mu(t_\iota,\psi^\iota)-2\mu(t_{\iota-1},\psi^{\iota-1})+\mu(t_{\iota-2},\psi^{\iota-2})}{2(\Delta t)^2}\int_{t_\iota}^{t_{\iota+1}}(\rho-t_{-2+\iota})(\rho-t_{-1+\iota})(t_{\iota+1}-\rho)^{\upsilon_1-1}d\rho.$$

As a result of computing the integrals in system (28), we now have

$$\int_{t_\iota}^{t_{\iota+1}}(t_{n+1}-\rho)^{\upsilon_1-1}d\rho = \frac{(\Delta t)^{\upsilon_1}}{\upsilon_1}\Big[(-\iota+m+1)^{\upsilon_1}-(-\iota+m)^{\upsilon_1}\Big]. \tag{29}$$

$$\int_{t_\iota}^{t_{\iota+1}}(\rho-t_{\iota-2})(t_{n+1}-\rho)^{\upsilon_1-1}d\rho = \frac{(\Delta t)^{\upsilon_1+1}}{\upsilon_1(\upsilon_1+1)}\Big[(-\iota+m+1)^{\upsilon_1}(-\iota+m+3+2\upsilon_1) - (-\iota+m+1)^{\upsilon_1}(-\iota+3+m+3\upsilon_1)\Big]. \tag{30}$$

$$\int_{t_\iota}^{t_{\iota+1}}(\rho-t_{\iota-2})(\rho-t_{\iota-1})(t_{n+1}-\rho)^{\upsilon_1-1}d\rho = \frac{(\Delta t)^{\upsilon_1+2}}{\upsilon_1(\upsilon_1+1)(\upsilon_1+2)}\Big[(m-\iota+1)^{\upsilon_1}\Big\{2(m-\iota)^2+(3\upsilon_1+10)(m-\iota)+2\upsilon_1^2+9\upsilon_1+12\Big\}-(m-\iota)^{\upsilon_1}\Big\{2(m-\iota)^2+(5\upsilon_1+10)(m-\iota)+6\upsilon_1^2+18\upsilon_1+12\Big\}\Big]. \tag{31}$$

Inserting them into system (28), we get

$$\psi^{n+1}(t) = \psi_0 + \frac{(1-\upsilon_1)\mu(t_n,\psi(t_n))}{AB(\upsilon_1)} + \frac{\upsilon_1(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+1)}\sum_{\iota=2}^{n}\mu(t_{\iota-2},\psi^{\iota-2})\Big[(n-\iota+1)^{\upsilon_1}$$
$$-(n-\iota)^{\upsilon_1}\Big] + \frac{\upsilon_1(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+2)}\sum_{\iota=2}^{n}\Big[\mu(t_{\iota-1},\psi^{\iota-1})-\mu(t_{\iota-2},\psi^{\iota-2})\Big]\Big[(n-\iota+1)^{\upsilon_1}(n-\iota+3+2\upsilon_1)$$
$$-(n-\iota)^{\upsilon_1}(n-\iota+3+3\upsilon_1)\Big] + \frac{\upsilon_1(\Delta t)^{\upsilon_1}}{2AB(\upsilon_1)\Gamma(\upsilon_1+3)}\sum_{\iota=2}^{n}\Big[\mu(t_\iota,\psi^\iota)-2\mu(t_{\iota-1},\psi^{\iota-1})+\mu(t_{\iota-2},\psi^{\iota-2})\Big]$$
$$\Big[(n-\iota+1)^{\upsilon_1}\Big\{2(n-\iota)^2+(3\upsilon_1+10)(n-\iota)+2\upsilon_1^2+9\upsilon_1+12\Big\}$$
$$-(n-\iota)^{\upsilon_1}\Big\{2(n-\iota)^2+(5\upsilon_1+10)(n-\iota)+6\upsilon_1^2+18\upsilon_1+12\Big\}\Big]. \tag{32}$$

The following numerical pattern may be produced by replacing $\mu(t,\psi(t)) = \Phi(t,\psi(t))\upsilon_2 t^{1-\upsilon_2}$.

$$\begin{aligned}
\psi^{n+1}(t) &= \psi_0 + \frac{(1-\upsilon_1)\Phi(t_n,\psi(t_n))\upsilon_2 t_n^{1-\upsilon_2}}{AB(\upsilon_1)} + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+1)}\sum_{\iota=2}^{n}\Phi(t_{\iota-2},\psi^{\iota-2})t_{\iota-2}^{1-\upsilon_2} \\
&\Big[(n-\iota+1)^{\upsilon_1}-(n-\iota)^{\upsilon_1}\Big] + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+2)}\sum_{\iota=2}^{n}\Big[\Phi(t_{\iota-1},\psi^{\iota-1})t_{\iota-1}^{1-\upsilon_2}-\Phi(t_{\iota-2},\psi^{\iota-2})t_{\iota-2}^{1-\upsilon_2}\Big] \\
&\Big[(n-\iota+1)^{\upsilon_1}(n-\iota+3+2\upsilon_1)-(n-\iota)^{\upsilon_1}(n-\iota+3+3\upsilon_1)\Big] + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{2AB(\upsilon_1)\Gamma(\upsilon_1+3)} \\
&\sum_{\iota=2}^{n}\Big[\Phi(t_\iota,\psi^\iota)t_\iota^{1-\upsilon_2}-2\Phi(t_{\iota-1},\psi^{\iota-1})t_{\iota-1}^{1-\upsilon_2}+\Phi(t_{\iota-2},\psi^{\iota-2})t_{\iota-2}^{1-\upsilon_2}\Big] \\
&\Big[(n-\iota+1)^{\upsilon_1}\Big\{2(n-\iota)^2+(3\upsilon_1+10)(n-\iota)+2\upsilon_1^2+9\upsilon_1+12\Big\} \\
&-(n-\iota)^{\upsilon_1}\Big\{2(n-\iota)^2+(5\upsilon_1+10)(n-\iota)+6\upsilon_1^2+18\upsilon_1+12\Big].
\end{aligned} \tag{33}$$

The following is what we get for system (5).

$$\begin{aligned}
\mathbf{S}^{n+1}(t) &= \mathbf{S}_0 + \frac{(1-\upsilon_1)\Phi(t_n,\mathbf{S}(t_n))\upsilon_2 t_n^{1-\upsilon_2}}{AB(\upsilon_1)} + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+1)}\sum_{\iota=2}^{n}\Phi(t_{\iota-2},\mathbf{S}^{\iota-2})t_{\iota-2}^{1-\upsilon_2} \\
&\Big[(n-\iota+1)^{\upsilon_1}-(n-\iota)^{\upsilon_1}\Big] + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+2)}\sum_{\iota=2}^{n}\Big[\Phi(t_{\iota-1},\mathbf{S}^{\iota-1})t_{\iota-1}^{1-\upsilon_2}-\Phi(t_{\iota-2},\mathbf{S}^{\iota-2})t_{\iota-2}^{1-\upsilon_2}\Big] \\
&\Big[(n-\iota+1)^{\upsilon_1}(n-\iota+3+2\upsilon_1)-(n-\iota)^{\upsilon_1}(n-\iota+3+3\upsilon_1)\Big] + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{2AB(\upsilon_1)\Gamma(\upsilon_1+3)} \\
&\sum_{\iota=2}^{n}\Big[\Phi(t_\iota,\mathbf{S}^\iota)t_\iota^{1-\upsilon_2}-2\Phi(t_{\iota-1},\mathbf{S}^{\iota-1})t_{\iota-1}^{1-\upsilon_2}+\Phi(t_{\iota-2},\mathbf{S}^{\iota-2})t_{\iota-2}^{1-\upsilon_2}\Big] \\
&\Big[(n-\iota+1)^{\upsilon_1}\Big\{2(n-\iota)^2+(3\upsilon_1+10)(n-\iota)+2\upsilon_1^2+9\upsilon_1+12\Big\} \\
&-(n-\iota)^{\upsilon_1}\Big\{2(n-\iota)^2+(5\upsilon_1+10)(n-\iota)+6\upsilon_1^2+18\upsilon_1+12\Big].
\end{aligned} \tag{34}$$

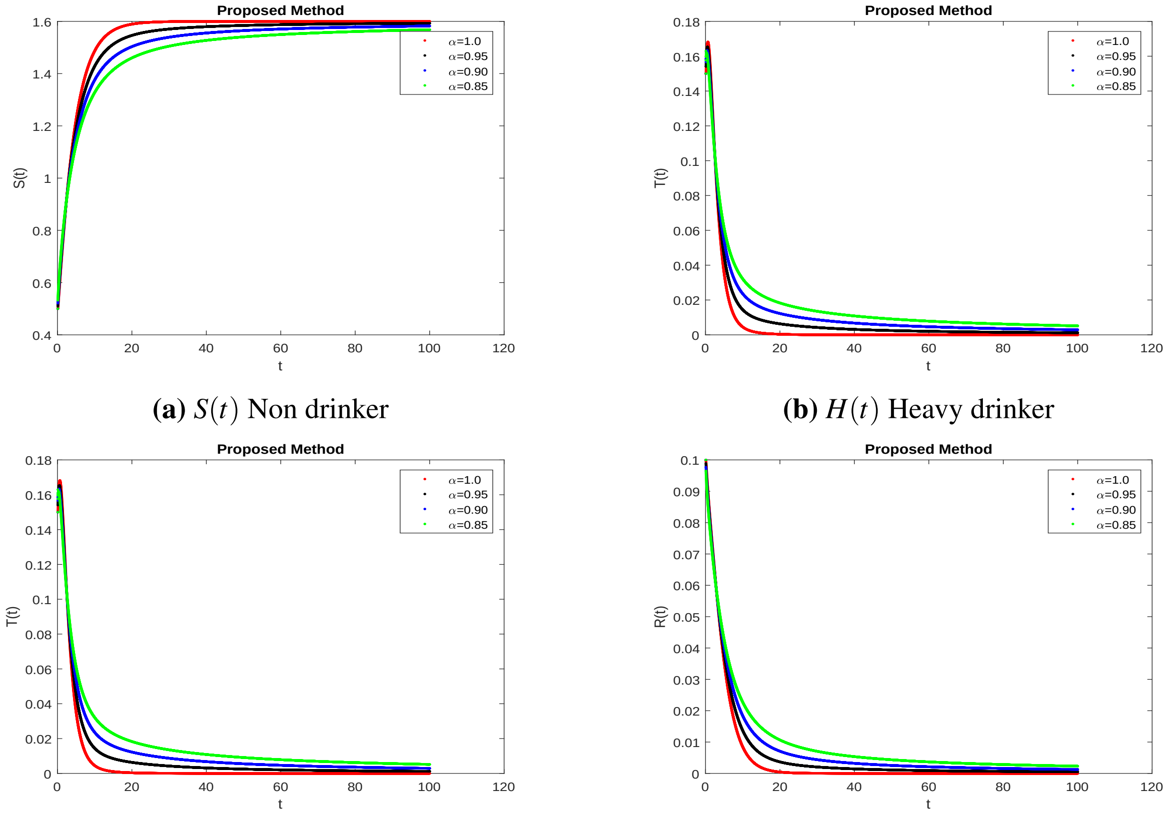
$$\begin{aligned}
\mathbf{H}^{n+1}(t) &= \mathbf{H}_0 + \frac{(1-\upsilon_1)\Phi(t_n,\mathbf{H}(t_n))\upsilon_2 t_n^{1-\upsilon_2}}{AB(\upsilon_1)} + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+1)}\sum_{\iota=2}^{n}\Phi(t_{\iota-2},\mathbf{H}^{\iota-2})t_{\iota-2}^{1-\upsilon_2} \\
&\Big[(n-\iota+1)^{\upsilon_1}-(n-\iota)^{\upsilon_1}\Big] + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+2)}\sum_{\iota=2}^{n}\Big[\Phi(t_{\iota-1},\mathbf{H}^{\iota-1})t_{\iota-1}^{1-\upsilon_2}-\Phi(t_{\iota-2},\mathbf{H}^{\iota-2})t_{\iota-2}^{1-\upsilon_2}\Big] \\
&\Big[(n-\iota+1)^{\upsilon_1}(n-\iota+3+2\upsilon_1)-(n-\iota)^{\upsilon_1}(n-\iota+3+3\upsilon_1)\Big] + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{2AB(\upsilon_1)\Gamma(\upsilon_1+3)} \\
&\sum_{\iota=2}^{n}\Big[\Phi(t_\iota,\mathbf{H}^\iota)t_\iota^{1-\upsilon_2}-2\Phi(t_{\iota-1},\mathbf{H}^{\iota-1})t_{\iota-1}^{1-\upsilon_2}+\Phi(t_{\iota-2},\mathbf{H}^{\iota-2})t_{\iota-2}^{1-\upsilon_2}\Big] \\
&\Big[(n-\iota+1)^{\upsilon_1}\Big\{2(n-\iota)^2+(3\upsilon_1+10)(n-\iota)+2\upsilon_1^2+9\upsilon_1+12\Big\} \\
&-(n-\iota)^{\upsilon_1}\Big\{2(n-\iota)^2+(5\upsilon_1+10)(n-\iota)+6\upsilon_1^2+18\upsilon_1+12\Big].
\end{aligned} \tag{35}$$

$$
\begin{aligned}
\mathbf{T}^{n+1}(t) &= \mathbf{T}_0 + \frac{(1-\upsilon_1)\Phi(t_n,\mathbf{T}(t_n))\upsilon_2 t_n^{1-\upsilon_2}}{AB(\upsilon_1)} + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+1)}\sum_{\iota=2}^{n}\Phi(t_{\iota-2},\mathbf{T}^{\iota-2})t_{\iota-2}^{1-\upsilon_2} \\
&\Big[(n-\iota+1)^{\upsilon_1}-(n-\iota)^{\upsilon_1}\Big] + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+2)}\sum_{\iota=2}^{n}\Big[\Phi(t_{\iota-1},\mathbf{T}^{\iota-1})t_{\iota-1}^{1-\upsilon_2}-\Phi(t_{\iota-2},\mathbf{T}^{\iota-2})t_{\iota-2}^{1-\upsilon_2}\Big] \\
&\Big[(n-\iota+1)^{\upsilon_1}(n-\iota+3+2\upsilon_1)-(n-\iota)^{\upsilon_1}(n-\iota+3+3\upsilon_1)\Big] + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{2AB(\upsilon_1)\Gamma(\upsilon_1+3)} \\
&\sum_{\iota=2}^{n}\Big[\Phi(t_\iota,\mathbf{T}^\iota)t_\iota^{1-\upsilon_2}-2\Phi(t_{\iota-1},\mathbf{T}^{\iota-1})t_{\iota-1}^{1-\upsilon_2}+\Phi(t_{\iota-2},\mathbf{T}^{\iota-2})t_{\iota-2}^{1-\upsilon_2}\Big] \\
&\Big[(n-\iota+1)^{\upsilon_1}\Big\{2(n-\iota)^2+(3\upsilon_1+10)(n-\iota)+2\upsilon_1^2+9\upsilon_1+12\Big\} \\
&-(n-\iota)^{\upsilon_1}\Big\{2(n-\iota)^2+(5\upsilon_1+10)(n-\iota)+6\upsilon_1^2+18\upsilon_1+12\Big].
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
\mathbf{R}^{n+1}(t) &= \mathbf{R}_0 + \frac{(1-\upsilon_1)\Phi(t_n,\mathbf{R}(t_n))\upsilon_2 t_n^{1-\upsilon_2}}{AB(\upsilon_1)} + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+1)}\sum_{\iota=2}^{n}\Phi(t_{\iota-2},\mathbf{R}^{\iota-2})t_{\iota-2}^{1-\upsilon_2} \\
&\Big[(n-\iota+1)^{\upsilon_1}-(n-\iota)^{\upsilon_1}\Big] + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{AB(\upsilon_1)\Gamma(\upsilon_1+2)}\sum_{\iota=2}^{n}\Big[\Phi(t_{\iota-1},\mathbf{R}^{\iota-1})t_{\iota-1}^{1-\upsilon_2}-\Phi(t_{\iota-2},\mathbf{R}^{\iota-2})t_{\iota-2}^{1-\upsilon_2}\Big] \\
&\Big[(n-\iota+1)^{\upsilon_1}(n-\iota+3+2\upsilon_1)-(n-\iota)^{\upsilon_1}(n-\iota+3+3\upsilon_1)\Big] + \frac{\upsilon_1\upsilon_2(\Delta t)^{\upsilon_1}}{2AB(\upsilon_1)\Gamma(\upsilon_1+3)} \\
&\sum_{\iota=2}^{n}\Big[\Phi(t_\iota,\mathbf{R}^\iota)t_\iota^{1-\upsilon_2}-2\Phi(t_{\iota-1},\mathbf{R}^{\iota-1})t_{\iota-1}^{1-\upsilon_2}+\Phi(t_{\iota-2},\mathbf{R}^{\iota-2})t_{\iota-2}^{1-\upsilon_2}\Big] \\
&\Big[(n-\iota+1)^{\upsilon_1}\Big\{2(n-\iota)^2+(3\upsilon_1+10)(n-\iota)+2\upsilon_1^2+9\upsilon_1+12\Big\} \\
&-(n-\iota)^{\upsilon_1}\Big\{2(n-\iota)^2+(5\upsilon_1+10)(n-\iota)+6\upsilon_1^2+18\upsilon_1+12\Big].
\end{aligned} \tag{37}
$$

6. Discussions and Numerical Simulations

Here, we offer numerical simulations to support the validity of the stable iterative strategy. Two initial value stages for each of the pandemic drinking model's four components. We simulate all compartment regarding obtained data at different fractional orders of υ_1 and dimension of υ_2. The vulnerable population that is drinking free equilibrium of starting value is represented in figures 1-2. All classes converge to the steady state point at the initial values. The previously indicated classes, which all converge rapidly at low order, were also simulated for various fractional orders. To determine the numerical simulation for the fractional order drinking model utilizing various fractional values with varying dimensions, Matlab coding is utilized. It is observed from figure 1a and 1b that non drinker start increasing after certain time due to decrease in heavy drinkers respectively. Treatment drinkers and recovered from drinker due to treatment start decreasing after certain time using different fractional values at dimension 1 as can be seen in figure 1c and 1d respectively. Similar behavior can be seen in figure 2a, 2b, 2c and 2d with minor effect of dimension using different fractional values at dimension 0.9. It is easily observed that Non drinkers rises and then remain at stable position after certain time due to treatment. It has been noted that academics can use this research to forecast future events and the kind of social impacts that would arise. In comparison to the traditional derivative, the fractal-fractional technique yields trustworthy results for every compartment based on the steady state at non-integer order derivatives.



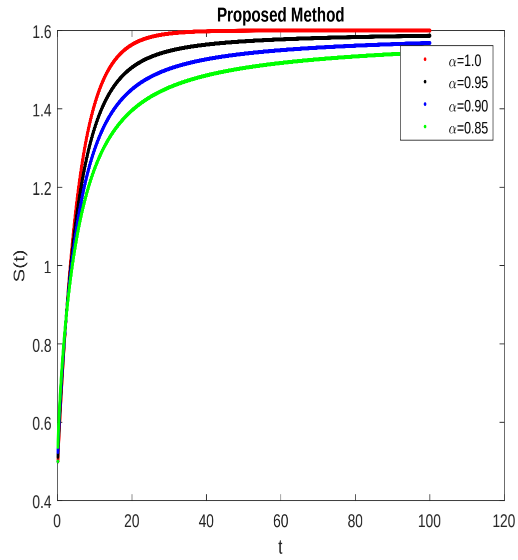
(a) $S(t)$ Non drinker

(b) $H(t)$ Heavy drinker

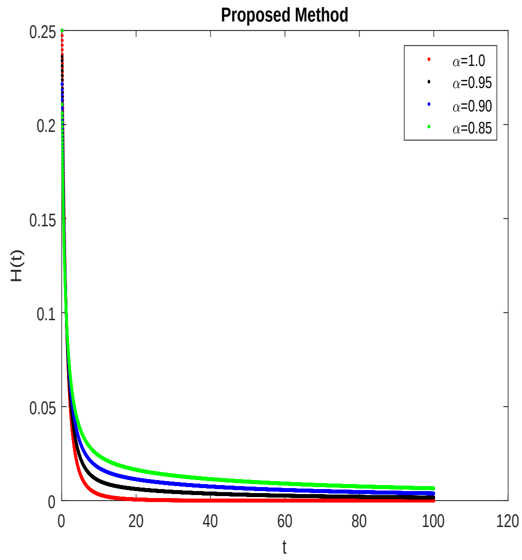
(c) $T(t)$ Drinker treatment

(d) $R(t)$ Recovered drinker

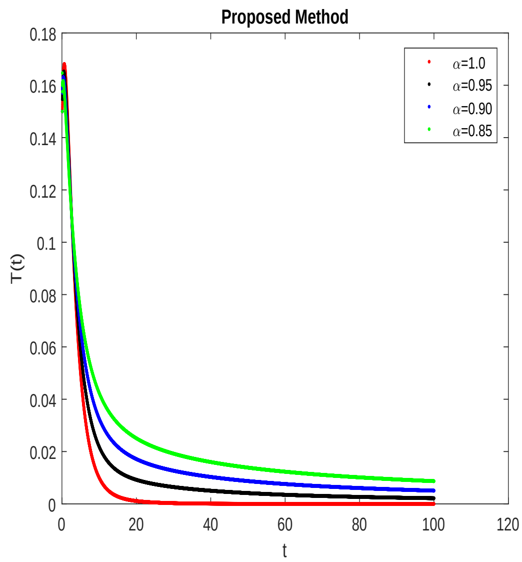
Figure 1: Compartments are simulated at dimension 1 using a fractal fractional operator.



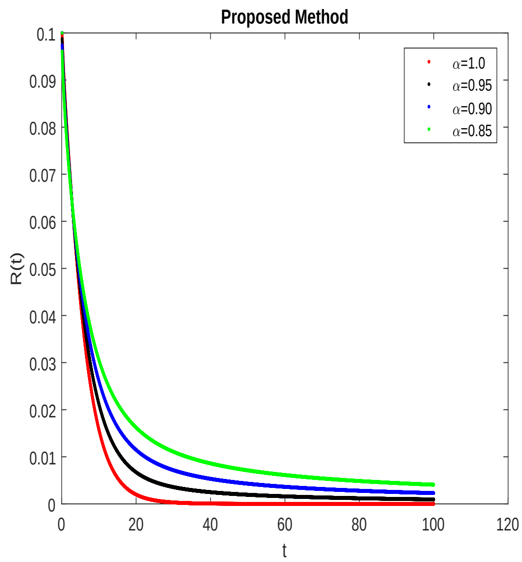
(a) $S(t)$ Non drinker



(b) $H(t)$ Heavy drinker



(c) $T(t)$ Drinker treatment



(d) $R(t)$ Recovered drinker

Figure 2: Compartments at dimension 0.9 are simulated using a fractal fractional operator.

7. Conclusion

The drinking triangle is affected by both internal and external stimuli, which illustrates the interaction between personalities. In this work, we presented fresh fractional derivatives of the Mittage-Leffler kernel that are closely related to one another. The fractional-order model exhibits many nonlinearities and influences on human mood. To comprehend various insights, the complicated dynamical behavior is explored from this connection. The set of fractional differential equations used to build drinker connections exhibits complicated and ambiguous real-world phenomena. Fractional order complex system is studied through the fractal fractional derivative with Mittage-Leffler kernel. With the use of fixed point theory, the fractional order system has verified the existence and singular solution. Additionally, check the system's stability to see how the two people's actual drinking histories affect each other. To illustrate the consequences of various fractional values that demonstrate actual behavior and the effects of real drinking stories, as well as to support theoretical conclusions. Numerical simulations are carried out for the proposed system in the range of fractional order to check the effect of fractional parameters. A deeper knowledge of the impact of love on a community will result from this type of study employing cutting-edge methods, and it will be useful in developing future measures to mitigate its negative impacts and enhance its beneficial benefits on society.

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