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Mathematical Modeling of Covid-19 Model dynamical transmission with different way of infections

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Abstract: The COVID-19 pandemic is causing a lot of pain on a global scale. This work aims to develop new mathematical models for the outbreak by utilising fractional derivatives. Through the control of certain diseases, the adoption of changed methodologies and fundamental explanations can have a substantial impact on the fitness of society. Examining the dynamics and numerical approximations for the suggested arbitrary-order coronavirus illness system is the primary goal. For the Covid-19 model, the study introduces fractional derivatives using sophisticated methods such as the Atangana-Baleanu, Sumudu transform, and Atangana-Toufik scheme. These methods yield accurate findings while examining the outbreak. Solutions involving various fractional operators are constructed using the Generalized Mittag-Leffler law. Covid-19 effects at various fractional values are explained and theoretical results are validated using numerical simulations. This aids in comprehending the outbreak and its management tactics.
Keywords: Fractional operators; Sumudu Transform; Qualitative analysis.

1. Introduction

The identification and analysis of an infection's behavior, as well as its screening or persistence in the community, are facilitated by a mathematical model [1]. Diseases caused by infectious viruses are spread by a variety of ways, including sneezing, coughing, and direct contact with people or water. Mathematical methods like as difference equations, initial circumstances, operating parameters, and statistical estimation are necessary for the analysis of these transmissions. These resources offer trustworthy epidemiological insights and efficient control methods [2]. With its first case in Wuhan, China, in December 2019, COVID-19, an acronym for coronavirus disease, started in 2019 and became a historic pandemic [3]. Then further, on 30th January of 2020, the World Health Organization exposed the bang that COVID-19 is a public health severe emergency and also labeled it as a huge pandemic on this planet on 11th March 2020. COVID-19 makes his directly to the respiratory system of humans, which is considered a severe respiratory (lungs) syndrome, i.e. corona virus 2 (SARS-CoV-2). Infected people either experience symptoms, or they don't even experience any. The COVID-19 spreads through physical interaction between individuals. The most common ratio of symptoms among the population is cough 68 per cent and fever 88 per cent which further change into shortening of breath (lungs) and pneumonia, and death at last. The virus usually takes 1-12 days to incubate, and the best ways to stop it from spreading are to wear masks, use sanitizer, and keep a distance of 2 meters. To overcome this disease totally, researchers worked on vaccines which were first given to old-aged people and then they were being offered, which included double dosages to a single person [4]. These vaccines bravely helped to slow the spread of COVID-19. This spread's primary target areas include workplaces, educational institutions, offices, marketplaces, and other public spaces [5]. This virus was spread rapidly and infected many humans, from which we had nearly four million confirmed cases in 187 countries. The figure of people who have lost their lives is more than 295,000, which results in the world's highest death toll [6]. Asian countries were also affected by this virus. In Pakistan, the very first case of COVID-19 was reported on 27th February 2020. And then this virus gets increases with the passage of time. WHO updated that from 3rd January 2020 to 7th October 2021, the confirmed infected cases are 1,253,868 and confirmed deaths are 27,986 all over Pakistan. The graph of infected people and death rates are gradually decreasing due to the use of vaccination.

First proposed by Leibniz in 1695, fractional derivatives are classified into three forms with a solitary kernel: Caputo, Riemann-Liouville, and Katugampola [7]. Researchers have also examined certain fractional order model applications [8]-[17]. When utilizing non-local and non-singular kernels to analyze the dynamic behavior of biological objects and systems, ABC (Mittag-Leffler) fractional derivatives [18] without singular kernels are useful tools. Fractional models of COVID-19 are examined in recent work by Atangana and Khan [19], with particular attention to China's circumstances during the pandemic. In our investigation, we comprehend the behavior of the virus, which first appeared in early 2020 and is currently unchecked. One tool for comprehending physical processes is fractional calculus. With many other researchers of our antecedents, we can clearly and confidently say that Fractional ordered derivative has been given the most accurate results as compared to integer order. This has been agreed upon in many research documents, books and monographs, as mentioned here [20]-[30].

In this thesis, we are concerned with the covid-19 modelling model. The corresponding derivative is taken in Atangana-Baleanu-Caputo and Atangana-Baleanu-Caputo with Toufik scheme sense. Qualitative analysis with positively and boundness of the proposed model is treated. Uniqueness, stability analysis of scheme established by using the fixed point theorem. In the end, numerical simulations are carried out of the fractional order COVID-19 model to check the memory effect of the fractional operator to overcome the risk of bad impact of disease on society.

2. Basic Concepts

Definition 2.1: The fractional-order derivative of ABC in Liouville-Caputo sense is mentioned as

$$ {}^{ABC}_{\gamma_1}D_t^{\gamma_1}\{f(t)\} = \frac{AB(\gamma_1)}{m-\gamma_1}\int_{\gamma_1}^{t}\frac{d^m}{dw^m}f(w)E_{\gamma_1}[-\gamma_1\frac{(t-w)^{\gamma_1}}{m-\gamma_1}]dw, m-1<\gamma_1<m, \tag{1}$$

where $AB(\gamma_1)$ is a normalization function $(AB(0)=AB(1)=1)$, and E_{γ_1} is the Mittag-Leffler function. The Atangana-Baleanu fractional integral is given by

$$ {}^{ABC}_{\gamma_1}I_t^{\gamma_1}\{f(t)\} = \frac{1-(\gamma_1)}{B-\gamma_1}f(t) + \frac{(\gamma_1)}{B(\gamma_1)\Gamma(\gamma_1)}\int_{\gamma_1}^{t}f(s)(t-s)^{\gamma_1-1}ds. \tag{2}$$

The Laplace transform of above is given by

$$ [{}^{ABC}_{\gamma_1}D_t^{\gamma_1}f(t)](s) = \frac{AB(\gamma_1)}{1-\gamma_1}\frac{s^{\gamma_1}L[f(t)](s)-s^{\gamma_1-1}f(0)}{s^{\gamma_1}+\frac{\gamma_1}{1-\gamma_1}}. \tag{3}$$

Definition 2.2: The Smudu transformation ($\mathfrak{W}$) is given as

$$ \mathfrak{W}[{}^{ABC}_{\gamma_1}D_t^{\gamma_1}f(t)](s) = \frac{B(\gamma_1)}{1-\gamma_1}(\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}v^{\gamma_1}))\times[\mathfrak{W}(f(t))-f(0)]. \tag{4}$$

3. Materials and Method

In this section, we consider the covid-19 model for further study. We have followings system of differential equations with fractional operator is given as

$$
\begin{aligned}
{}_{0}^{ABC}D_t^{\gamma_1} S(t) &= -S(t)(\alpha_1 I(t) + \beta_1 D(t) + \gamma_1 A(t) + \delta_1 R(t)),\\
{}_{0}^{ABC}D_t^{\gamma_1} I(t) &= S(t)(\alpha_1 I(t) + \beta_1 D(t) + \gamma_1 A(t) + \delta_1 R(t)) - (\varepsilon_1 + \zeta_1 + \lambda_1) I(t),\\
{}_{0}^{ABC}D_t^{\gamma_1} D(t) &= \varepsilon_1 I(t) - (\eta_1 + \rho_1) D(t),\\
{}_{0}^{ABC}D_t^{\gamma_1} A(t) &= \zeta_1 I(t) - (\theta_1 + \mu_1 + \kappa_1) A(t),\\
{}_{0}^{ABC}D_t^{\gamma_1} R(t) &= \eta_1 D(t) + \theta_1 A(t) - (\nu_1 + \xi_1) R(t),\\
{}_{0}^{ABC}D_t^{\gamma_1} T(t) &= \mu_1 A(t) + \nu_1 R(t) - (\sigma_1 + \tau_1) T(t),\\
{}_{0}^{ABC}D_t^{\gamma_1} H(t) &= \lambda_1 I(t) + \rho_1 D(t) + \kappa_1 A(t) + \xi_1 R(t) + \sigma_1 T(t),
\end{aligned}
$$

$$
{}_{0}^{ABC}D_t^{\gamma_1} E(t) = \tau_1 T(t), \tag{5}
$$

with

$$
S_0(t) = S(0),\ \ I_0(t) = I(0),\ \ D_0(t) = D(0),\ \ A_0(t) = A(0),
$$

$$
R_0(t) = R(0),\ \ T_0(t) = T(0),\ \ H_0(t) = H(0),\ \ E_0(t) = E(0), \tag{6}
$$

where S(t) shows susceptible people, I(t) shows infected undetected people, D(t) shows confirmed infected people, A(t) shows ailing symptomatic infected people, R(t) shows recognized infected people, T(t) shows recognized acutely symptoms infected people, H(t) shows healed people and E(t) shows death people.

From Sumudu Transform operator, we have

$$
\begin{aligned}
&\frac{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)}{(1-\gamma_1)} E_{\gamma_1}\Big(-\frac{1}{1-\gamma_1}\omega^{\gamma_1}\Big)[ST(S(t)) - S(0)]\\
&= \mathfrak{W}[-S(t)(\alpha_1 I(t) + \beta_1 D(t) + \gamma_1 A(t) + \delta_1 R(t))],\\
&\frac{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)}{(1-\gamma_1)} E_{\gamma_1}\Big(-\frac{1}{1-\gamma_1}\omega^{\gamma_1}\Big)[ST(I(t)) - I(0)]\\
&= \mathfrak{W}[S(t)(\alpha_1 I(t) + \beta_1 D(t) + \gamma_1 A(t) + \delta_1 R(t)) - (\varepsilon_1 + \zeta_1 + \lambda_1) I(t)],\\
&\frac{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)}{(1-\gamma_1)} E_{\gamma_1}\Big(-\frac{1}{1-\gamma_1}\omega^{\gamma_1}\Big)[ST(D(t)) - D(0)]\\
&= \mathfrak{W}[\varepsilon_1 I(t) - (\eta_1 + \rho_1) D(t)],\\
&\frac{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)}{(1-\gamma_1)} E_{\gamma_1}\Big(-\frac{1}{1-\gamma_1}\omega^{\gamma_1}\Big)[ST(A(t)) - A(0)]\\
&= \mathfrak{W}[\zeta_1 I(t) - (\theta_1 + \mu_1 + \kappa_1) A(t)],\\
&\frac{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)}{(1-\gamma_1)} E_{\gamma_1}\Big(-\frac{1}{1-\gamma_1}\omega^{\gamma_1}\Big)[ST(R(t)) - R(0)]\\
&= \mathfrak{W}[\eta_1 D(t) + \theta_1 A(t) - (\nu_1 + \xi_1) R(t)],\\
&\frac{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)}{(1-\gamma_1)} E_{\gamma_1}\Big(-\frac{1}{1-\gamma_1}\omega^{\gamma_1}\Big)[ST(T(t)) - T(0)]\\
&= \mathfrak{W}[\mu_1 A(t) + \nu_1 R(t) - (\sigma_1 + \tau_1) T(t)],
\end{aligned}
$$

$$\frac{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)}{(1-\gamma_1)}E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})[ST(H(t))-H(0)]$$
$$=\mathfrak{W}[\lambda_1 I(t)+\rho_1 D(t)+\kappa_1 A(t)+\xi_1 R(t)+\sigma_1 T(t)],$$
$$\frac{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)}{(1-\gamma_1)}E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})[ST(E(t))-E(0)]$$
$$=\mathfrak{W}[\tau_1 T(t)]. \tag{7}$$

We get

$$ST(S(t))=S(0)+\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[-S(t)(\alpha_1 I(t)+\beta_1 D(t)+\gamma_1 A(t)+\delta_1 R(t))],$$
$$ST(I(t))=I(0)+\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[S(t)(\alpha_1 I(t)+\beta_1 D(t)+\gamma_1 A(t)+\delta_1 R(t))-(\varepsilon_1+\zeta_1+\lambda_1)I(t)],$$
$$ST(D(t))=D(0)+\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\varepsilon_1 I(t)-(\eta_1+\rho_1)D(t)],$$
$$ST(A(t))=A0)+\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\zeta_1 I(t)-(\theta_1+\mu_1+\kappa_1)A(t)],$$
$$ST(R(t))=R(0)+\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\eta_1 D(t)+\theta_1 A(t)-(\nu_1+\xi_1)R(t)],$$
$$ST(T(t))=T(0)+\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\mu_1 A(t)+\nu_1 R(t)-(\sigma_1+\tau_1)T(t)],$$
$$ST(H(t))=H(0)+\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\lambda_1 I(t)+\rho_1 D(t)+\kappa_1 A(t)+\xi_1 R(t)+\sigma_1 T(t)],$$
$$ST(E(t))=E(0)+\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\tau_1 T(t)], \tag{8}$$

By using inverse Sumudu Transform, we find

$$S(t)=S(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[-S(t)(\alpha_1 I(t)+\beta_1 D(t)+\gamma_1 A(t)+\delta_1 R(t))]\},$$
$$I(t)=I(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[S(t)(\alpha_1 I(t)+\beta_1 D(t)+\gamma_1 A(t)+\delta_1 R(t))-(\varepsilon_1+\zeta_1+\lambda_1)I(t)]\},$$

$$D(t)=D(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\varepsilon_1 I(t)-(\eta_1+\rho_1)D(t)]\},$$

$$A(t)=A(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\zeta_1 I(t)-(\theta_1+\mu_1+\kappa_1)A(t)]\},$$

$$R(t)=R(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\eta_1 D(t)+\theta_1 A(t)-(\nu_1+\xi_1)R(t)]\},$$

$$T(t)=T(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\mu_1 A(t)+\nu_1 R(t)-(\sigma_1+\tau_1)T(t)]\},$$

$$H(t)=H(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\lambda_1 I(t)+\rho_1 D(t)+\kappa_1 A(t)+\xi_1 R(t)+\sigma_1 T(t)]\},$$

$$E(t)=E(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\tau_1 T(t)]\}. \tag{9}$$

Thus, we get finally

$$S_{(m+1)}(t)=S_m(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[-S_m(t)(\alpha_1 I_m(t)+\beta_1 D_m(t)+\gamma_1 A_m(t)+\delta_1 R_m(t))]\},$$

$$I_{(m+1)}(t)=I_m(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[S_m(t)(\alpha_1 I_m(t)+\beta_1 D_m(t)+\gamma_1 A_m(t)+\delta_1 R_m(t))-(\varepsilon_1+\zeta_1+\lambda_1)I_m(t)]\},$$

$$D_{(m+1)}(t)=D_m(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\varepsilon_1 I_m(t)-(\eta_1+\rho_1)D_m(t)]\},$$

$$A_{(m+1)}(t)=A_m(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\zeta_1 I_m(t)-(\theta_1+\mu_1+\kappa_1)A_m(t)]\},$$

$$R_{(m+1)}(t)=R_m(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\eta_1 D_m(t)+\theta_1 A_m(t)-(\nu_1+\xi_1)R_m(t)]\},$$

$$T_{(m+1)}(t)=T_m(0)+\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\mu_1 A_m(t)+\nu_1 R_m(t)-(\sigma_1+\tau_1)T_m(t)]\},$$

$$H_{(m+1)}(t) = H_m(0) + \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\lambda_1 I_m(t) + \rho_1 D_m(t) + \kappa_1 A_m(t) + \xi_1 R_m(t) + \sigma_1 T_m(t)]\},$$
$$E_{(m+1)}(t) = E_m(0) + \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\tau_1 T_m(t)]\}. \tag{10}$$

Theorem 1:
Let H is a self-map of a Banach space $(X, |.|)$ such that

$$\|H_r - H_x\| \leq \theta\|X - H_r\| + \theta\|r - x\|, \quad r, x\varepsilon X, \quad \theta \in (0,1]. \tag{11}$$

Therefore, H is Picard H-stable.
From Equation (10), we have

$$\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})} \tag{12}$$

Proof
Assume K is a self-map defined by

$$K[S_{(m+1)}] = S_m(0) + \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[-S_m(t)(\alpha_1 I_m(t) + \beta_1 D_m(t) + \gamma_1 A_m(t) + \delta_1 R_m(t))]$$
$$K[I_{(m+1)}] = I_m(0) + \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[S_m(t)(\alpha_1 I_m(t) + \beta_1 D_m(t) + \gamma_1 A_m(t) + \delta_1 R_m(t)) - (\varepsilon_1 + \zeta_1 + \lambda_1) I_m]$$
$$K[D_{(m+1)}] = D_m(0) + \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\varepsilon_1 I_m(t) - (\eta_1 + \rho_1) D_m]$$
$$K[A_{(m+1)}] = A_m(0) + \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\zeta_1 I_m(t) - (\theta_1 + \mu_1 + \kappa_1) A_m]$$
$$K[R_{(m+1)}] = R_m(0) + \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\eta_1 D_m(t) + \theta_1 A_m(t) - (\nu_1 + \xi_1) R_m]$$
$$K[T_{(m+1)}] = T_m(0) + \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\mu_1 A_m(t) + \nu_1 R_m(t) - (\sigma_1 + \tau_1) T_m]$$
$$K[H_{(m+1)}] = H_m(0) + \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$
$$\times\mathfrak{W}[\lambda_1 I_m(t) + \rho_1 D_m(t) + \kappa_1 A_m(t) + \xi_1 R_m(t) + \sigma_1 T_m]$$

$$K[E_{(m+1)}] = E_m(0) + \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\tau_1 T_m] \tag{13}$$

By using the norm's characteristics and accounting for the triangle inequality, we obtain

$$\|K[S_m]-K[S_n]\| \leq \|S_m(t)-S_n(t)\| + \|\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[-S_m(t)(\alpha_1 I_m(t)+\beta_1 D_m(t)+\gamma_1 A_m(t)+\delta_1 R_m(t))]\}$$

$$-\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[-S_n(t)(\alpha_1 I_n(t)+\beta_1 D_n(t)+\gamma_1 A_n(t)+\delta_1 R_n(t))]\}\|$$

$$\|K[I_m]-K[I_n]\| \leq \|E_m(t)-E_n(t)\| + \|\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[S_m(t)(\alpha_1 I_m(t)+\beta_1 D_m(t)+\gamma_1 A_m(t)+\delta_1 R_m(t))-(\varepsilon+\zeta+\lambda)I_m]\}$$

$$-\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[S_n(t)(\alpha I_n(t)+\beta D_n(t)+\gamma A_n(t)+\delta R_n(t))-(\varepsilon+\zeta+\lambda)I_n]\}$$

$$\|K[D_m]-K[D_n]\| \leq \|I_m(t)-I_n(t)\| + \|\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\varepsilon_1 I_m(t)-(\eta_1+\rho_1)D_m]\} - \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\varepsilon_1 I_n(t)-(\eta_1+\rho_1)D_n]\}\|$$

$$\|K[A_m]-K[A_n]\| \leq \|Q_m(t)-Q_n(t)\| + \|\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\zeta_1 I_m(t)-(\theta_1+\mu_1+\kappa_1)A_m]\} - \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\zeta_1 I_n(t)-(\theta_1+\mu_1+\kappa_1)A_n]\}$$

$$\|K[R_m]-K[R_n]\| \leq \|R_m(t)-R_n(t)\| + \|\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\eta_1 D_m(t)+\theta_1 A_m(t)-(\nu_1+\xi_1)R_m]\} - \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\eta_1 D_n(t)+\theta_1 A_n(t)-(\nu_1+\xi_1)R_n]\}\|$$

$$\|K[T_m]-K[T_n]\| \leq \|R_m(t)-R_n(t)\| + \|\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\mu_1 A_m(t)+\nu_1 R_m(t)-(\sigma_1+\tau_1)T_m]\} - \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\mu_1 A_n(t)+\nu_1 R_n(t)-(\sigma_1+\tau_1)T_n]\}\|$$

$$\|K[H_m]-K[H_n]\| \leq \|R_m(t)-R_n(t)\| + \|\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\lambda_1 I_m(t)+\rho_1 D_m(t)+\kappa_1 A_m(t)+\xi_1 R_m(t)+\sigma_1 T_m]\} - \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\lambda_1 I_n(t)+\rho_1 D_n(t)+\kappa_1 A_n(t)+\xi R_n(t)+\sigma_1 T_n]\}\|$$

$$\|K[E_m]-K[E_n]\| \leq \|P_m(t)-P_n(t)\| + \|\mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\tau_1 T_m]\} - \mathfrak{W}^{-1}\{\frac{1-\gamma_1}{B(\gamma_1)\gamma_1\Gamma(\gamma_1+1)E_{\gamma_1}(-\frac{1}{1-\gamma_1}\omega^{\gamma_1})}$$

$$\times\mathfrak{W}[\tau_1 T_n(t)]\} \tag{14}$$

Under the condition:

$$\theta = \begin{cases} \|S_m-S_n\| \times \|-S_m+S_n\| - \|S_m-S_n\| \\ (\alpha_1\|I_m-I_n\| + \beta_1\|D_m-D_n\| + \gamma_1\|A_m-A_n\| \\ +\delta\|R_m-R_n\|, \\ \times\|I_m-I_n\| \times \|-I_m+I_n\| + \|S_m-S_n\| \\ (\alpha_1\|I_m-I_n\| + \beta_1\|D_m-D_n\| + \gamma_1\|A_m-A_n\| \\ +\delta_1\|R_m-R_n\| - (\varepsilon_1+\zeta_1+\lambda_1)\|I_m-I_n\| \\ \times\|D_m-D_n\| \times \|-D_m+D_n\| + \varepsilon_1\|I_m-I_n\| \\ -(\eta_1+\rho_1)\|D_m-D_n\|, \\ \times\|A_m-A_n\| \times \|-A_m+A_n\| + \zeta_1\|I_m-I_n\| \\ -(\theta_1+\mu_1+\kappa_1)\|A_m-A_n\|, \\ \times\|R_m-R_n\| \times \|-R_m+R_n\| + \eta_1\|D_m-D_n\| \\ +\theta_1\|A_m-A_n\| - (\nu_1+\xi_1)\|R_m-R_n\| \\ \times\|T_m-T_n\| \times \|-T_m+T_n\| + \mu_1\|A_m-A_n\| \\ +\nu_1\|R_m-R_n\| - (\sigma_1+\tau_1)\|T_m-T_n\| \\ \times\|H_m-H_n\| \times \|-H_m+H_n\| + \lambda_1\|I_m-I_n\| \\ +\rho_1\|D_m-D_n\| + \kappa_1\|A_m-A_n\| + \xi_1\|R_m-R_n\| \\ +\sigma_1\|T_m-T_n\| \\ \times\|E_m-E_n\| \times \|-E_m+E_n\| + \tau_1\|T_m-T_n\|, \end{cases} \tag{15}$$

the stability of K is true.

4. Numerical Scheme With Atangana Toufik Apporach

Here we will use fractional derivative with non singular kernel and non local for constructing advance scheme for the equations of nonlinear fractional differential given in (5) using fractional parameter α. in this regard, we let these ordinary nonlinear fractional equation.

We obtain the following for system

$$S(t)-S(0) = \frac{(1-\alpha_2)}{ABC(\alpha_2)}\{-S(t)(\alpha_1 I(t)+\beta_1 D(t)+\gamma_1 A(t)+\delta_1 R(t))\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\int_0^t \{-S(\tau_1)(\alpha_1 I(\tau_1)+\beta D(\tau_1)+\gamma_1 A(\tau_1)+\delta_1 R(\tau_1))\}$$

$$(t-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$I(t)-I(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{S(t)(\alpha_1 I(t)+\beta_1 D(t)+\gamma_1 A(t)+\delta_1 R(t))-(\varepsilon_1+\zeta_1+\lambda_1)I(t)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\int_0^t\{S(\tau_1)(\alpha_2 I(\tau_1)+\beta_1 D(\tau_1)+\gamma_1 A(\tau_1)+\delta R(\tau_1))-(\varepsilon_1+\zeta_1+\lambda_1)I(\tau_1)\}$$

$$(t-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$D(t)-D(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\varepsilon_1 I(t)-(\eta_1+\rho_1)D(t)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\int_0^t\{\varepsilon I(\tau_1)-(\eta_1+\rho_1)D(\tau_1)\}$$

$$(t-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$A(t)-A(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\zeta_1 I(t)-(\theta_1+\mu_1+\kappa_1)A(t)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\int_0^t\{\zeta I(\tau_1)-(\theta_1+\mu_1+\kappa_1)A(\tau_1)\}$$

$$(t-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$R(t)-R(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\eta_1 D(t)+\theta_1 A(t)-(\nu_1+\xi_1)R(t)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\int_0^t\{\eta D(\tau_1)+\theta A(\tau_1)-(\nu_1+\xi_1)R(\tau_1)\}$$

$$(t-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$T(t)-T(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\mu_1 A(t)+\nu_1 R(t)-(\sigma_1+\tau_1)T(t)\}$$

$$+\frac{\alpha_1}{\Gamma(\alpha_1)\times ABC(\alpha_2)}\int_0^t\{\mu_1 A(\tau_1)+\nu_1 R(\tau_1)-(\sigma_1+\tau_1)T(\tau_1)\}$$

$$(t-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$H(t)-H(0)=\frac{(1-\alpha_1)}{ABC(\alpha_2)}\{\lambda_1 I(t)+\rho_1 D(t)+\kappa_1 A(t)+\xi_1 R(t)+\sigma_1 T(t)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_1)}\int_0^t\{\lambda_1 I(\tau_1)+\rho_1 D(\tau_1)+\kappa_1 A(\tau_1)+\xi_1 R(\tau_1)+\sigma_1 T(\tau_1)\}$$

$$(t-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$E(t)-E(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\tau T(t)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\int_0^t\{\tau_1 T(\tau_1)\}$$

$$(t-\tau_1)^{\alpha_1-1}d\tau_1,$$

At a given $t_{m+1}, m=0,1,2,3...$ the above equation is reformulated as

$$S(t_{m+1})-S(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{-S(t_m)(\alpha_1 I(t_m)+\beta_1 D(t_m)+\gamma_1 A(t_m)+\delta_1 R(t_m))\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}\int_{t_{k_1}}^{t_{k_1+1}}\{-S(\tau_1)(\alpha_1 I(\tau_1)+\beta_1 D(\tau_1)+\gamma_1 A(\tau_1)+\delta_1 R(\tau_1))\}$$

$$(t_{m+1}-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$I(t_{m+1})-I(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{S(t_m)(\alpha_1 I(t_m)+\beta_1 D(t_m)+\gamma_1 A(t_m)+\delta_1 R(t_m))-(\varepsilon_1+\zeta_1+\lambda_1)I(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}\int_{t_{k_1}}^{t_{k_1+1}}\{S(\tau_1)(\alpha_1 I(\tau_1)+\beta_1 D(\tau_1)+\gamma_1 A(\tau_1)+\delta_1 R(\tau_1))-(\varepsilon_1+\zeta_1+\lambda_1)I(\tau_1)\}$$

$$(t_{m+1}-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$D(t_{m+1})-D(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\varepsilon_1 I(t_m)-(\eta_1+\rho_1)D(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}\int_{t_{k_1}}^{t_{k_1+1}}\{\varepsilon_1 I(\tau_1)-(\eta_1+\rho_1)D(\tau_1)\}$$

$$(t_{m+1}-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$A(t_{m+1})-A(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\zeta_1 I(t_m)-(\theta_1+\mu_1+\kappa_1)A(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}\int_{t_{k_1}}^{t_{k_1+1}}\{\zeta_1 I(\tau_1)-(\theta_1+\mu_1+\kappa_1)A(\tau_1)\}$$

$$(t_{m+1}-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$R(t_{m+1})-R(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\eta_1 D(t_m)+\theta_1 A(t_m)-(\nu_1+\xi_1)R(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}\int_{t_{k_1}}^{t_{k_1+1}}\{\eta_1 D(\tau_1)+\theta_1 A(\tau_1)-(\nu_1+\xi_1)R(\tau_1)\}$$

$$(t_{m+1}-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$T(t_{m+1})-T(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\mu_1 A(t_m)+\nu_1 R(t_m)-(\sigma_1+\tau_1)T(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}\int_{t_{k_1}}^{t_{k_1+1}}\{\mu_1 A(\tau_1)+\nu_1 R(\tau_1)-(\sigma_1+\tau_1)T(\tau_1)\}$$

$$(t_{m+1}-\tau_1)^{\alpha_2-1}d\tau_1.$$

$$H(t_{m+1})-H(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\lambda_1 I(t_m)+\rho_1 D(t_m)+\kappa_1 A(t_m)+\xi_1 R(t_m)+\sigma_1 T(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}\int_{t_{k_1}}^{t_{k_1+1}}\{\lambda_1 I(\tau_1)+\rho_1 D(\tau_1)+\kappa_1 A(\tau_1)+\xi_1 R(\tau_1)+\sigma_1 T(\tau_1)\}$$

$$(t_{m+1}-\tau_1)^{\alpha_2-1}d\tau_1,$$

$$E(t_{m+1})-E(0)=\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\tau_1 T(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}\int_{t_{k_1}}^{t_{k_1+1}}\{\tau_1 T(\tau_1)\}$$

$$(t_{m+1}-\tau_1)^{\alpha_2-1}d\tau_1,$$

By using Equation, we have

$$S_{m+1} = S_0 + \frac{(1-\alpha_2)}{ABC(\alpha_2)}\{-S(t_m)(\alpha_1 I(t_m) + \beta_1 D(t_m) + \gamma_1 A(t_m) + \delta_1 R(t_m))\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{-S_{k_1}(\alpha_1 I_{k_1} + \beta_1 D_{k_1} + \gamma_1 A_{k_1} + \delta_1 R_{k_1})}{h}B_1$$

$$-\frac{-S_{k_1-1}(\alpha_1 I_{k_1-1} + \beta_1 D_{k_1-1} + \gamma_1 A_{k_1-1} + \delta_1 R_{k_1-1}}{h}A_{\alpha_2,k_1,2}),$$

$$I_{m+1} = I_0 + \frac{(1-\alpha_2)}{ABC(\alpha_1)}\{S(t_m)(\alpha_1 I(t_m) + \beta_1 D(t_m) + \gamma_1 A(t_m) + \delta_1 R(t_m)) - (\varepsilon_1 + \zeta_1 + \lambda_1)I(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{S_{k_1}(\alpha_1 I_{k_1} + \beta_1 D_{k_1} + \gamma_1 A_{k_1} + \delta_1 R_{k_1}) - (\varepsilon_1 + \zeta_1 + \lambda_1)I_{k_1}}{h}B_1$$

$$-\frac{S_{k_1-1}(\alpha_1 I_{k_1-1} + \beta_1 D_{k_1-1} + \gamma_1 A_{k_1-1} + \delta_1 R_{k_1-1} - (\varepsilon_1 + \zeta_1 + \lambda_1)I_{k_1-1}}{h}A_{\alpha_1,k_1,2}),$$

$$D_{m+1} = D_0 + \frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\varepsilon_1 I(t_m) - (\eta_1 + \rho_1)D(t_m)\}$$

$$+\frac{\alpha_1}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{\varepsilon_1 I_{k_1} - (\eta_1 + \rho_1)D_{k_1}}{h}B_1$$

$$-\frac{\varepsilon_1 I_{k_1-1} - (\eta_1 + \rho_1)D_{k_1-1}}{h}A_{\alpha_1,k_1,2}),$$

$$A_{m+1} = A_0 + \frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\zeta_1 I(t_m) - (\theta_1 + \mu_1 + \kappa_1)A(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{\zeta_1 I_{k_1} - (\theta_1 + \mu_1 + \kappa_1)A_{k_1}}{h}B_1$$

$$-\frac{\zeta_1 I_{k_1-1} - (\theta_1 + \mu_1 + \kappa_1)A_{k_1-1}}{h}A_{\alpha_1,k_1,2}),$$

$$R_{m+1} = R_0 + \frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\eta_1 D(t_m) + \theta_1 A(t_m) - (\nu_1 + \xi_1)R(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{\eta_1 D_{k_1} + \theta_1 A_{k_1} - (\nu_1 + \xi_1)R_{k_1}}{h}B_1$$

$$-\frac{\eta_1 D_{k_1-1} + \theta_1 A_{k_1-1} - (\nu_1 + \xi_1)R_{k_1-1}}{h}A_{\alpha_1,k_1,2}),$$

$$T_{m+1} = T_0 + \frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\mu A(t_m) + \nu R(t_m) - (\sigma + \tau)T(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_1)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{\mu_1 A_{k_1} + \nu_1 R_{k_1} - (\sigma_1 + \tau_1)T_{k_1}}{h}B_1$$

$$-\frac{\mu_1 A_{k_1-1} + \nu_1 R_{k_1-1} - (\sigma_1 + \tau_1)T_{k_1-1}}{h}A_{\alpha_1,k_1,2}),$$

$$H_{m+1} = H_0 + \frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\lambda_1 I(t_m) + \rho_1 D(t_m) + \kappa_1 A(t_m) + \xi_1 R(t_m) + \sigma_1 T(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_1)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{\lambda_1 I_{k_1}+\rho_1 D_{k_1}+\kappa_1 A_{k_1}+\xi_1 R_{k_1}+\sigma_1 T_{k_1}}{h}B_1$$

$$-\frac{\lambda_1 I_{k_1-1}+\rho_1 D_{k_1-1}+\kappa_1 A_{k_1-1}+\xi_1 R_{k_1-1}+\sigma_1 T_{k_1-1}}{h}A_{\alpha_1,k_1,2}),$$

$$E_{m+1}=E_0+\frac{(1-\alpha_2)}{ABC(\alpha_1)}\{\tau_1 T(t_m)\}$$

$$+\frac{\alpha_2}{\Gamma(\alpha_2)\times ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{\tau_1 T_{k_1}}{h}B_1$$

$$-\frac{\tau T_{k_1-1}}{h}A_{\alpha_2,k_1,2}),$$

where $A_{\alpha_2,k_1,2}=\int_{t_{k_1}}^{t_{k_1+1}}(\tau_1-t_{k_1})(t_{m+1}-\tau_1)^{\alpha_2-1}d\tau_1$ and $B_1=\int_{t_{k_1}}^{t_{k_1+1}}(\tau_1-t_{k_1-1})(t_{m+1}-\tau_1)^{\alpha_2-1}d\tau_1$. Thus, integrating Equations and replacing them in equations of system, we get

$$S_{m+1}=S_0+\frac{(1-\alpha_1)}{ABC(\alpha_2)}\{-S(t_m)(\alpha_1 I(t_m)+\beta_1 D(t_m)+\gamma_1 A(t_m)+\delta_1 R(t_m))\}$$

$$+\frac{\alpha_1}{ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{h^{\alpha_2}\{-S_{k_1}(\alpha_1 I_{k_1}+\beta_1 D_{k_1}+\gamma_1 A_{k_1}+\delta_1 R_{k_1})\}}{h}B_1$$

$$-\frac{h^{\alpha_2}\{-S_{k_1-1}(\alpha_1 I_{k_1-1}+\beta_1 D_{k_1-1}+\gamma_1 A_{k_1-1}+\delta_1 R_{k_1-1})\}}{h}A_{\alpha_1,k_1,2}),$$

$$I_{m+1}=I_0+\frac{(1-\alpha_2)}{ABC(\alpha_1)}\{S(t_m)(\alpha_1 I(t_m)+\beta_1 D(t_m)+\gamma_1 A(t_m)+\delta_1 R(t_m))-(\varepsilon_1+\zeta_1+\lambda_1)I(t_m)\}$$

$$+\frac{\alpha_1}{ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{h^{\alpha_2}\{S_{k_1}(\alpha_1 I_{k_1}+\beta_1 D_{k_1}+\gamma_1 A_{k_1}+\delta_1 R_{k_1})-(\varepsilon_1+\zeta_1+\lambda_1)I_{k_1}\}}{h}B_1$$

$$-\frac{h^{\alpha_2}\{S_{k_1-1}(\alpha_1 I_{k_1-1}+\beta_1 D_{k_1-1}+\gamma_1 A_{k_1-1}+\delta_1 R_{k_1-1})-(\varepsilon_1+\zeta_1+\lambda_1)I_{k_1-1}\}}{h}A_{\alpha_1,k_1,2}),$$

$$D_{m+1}=D_0+\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\varepsilon_1 I(t_m)-(\eta_1+\rho_1)D(t_m)\}$$

$$+\frac{\alpha_1}{ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{h^{\alpha_2}\{\varepsilon_1 I_{k_1}-(\eta_1+\rho_1)D_{k_1}\}}{h}B_1$$

$$-\frac{h^{\alpha_2}\{\varepsilon_1 I_{k_1-1}-(\eta_1+\rho_1)D_{k_1-1}\}}{h}A_{\alpha_2,k_1,2}),$$

$$A_{m+1}=A_0+\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\zeta_1 I(t_m)-(\theta_1+\mu_1+\kappa_1)A(t_m)\}$$

$$+\frac{\alpha_2}{ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{h^{\alpha_2}\{\zeta I_{k_1}-(\theta_1+\mu_1+\kappa_1)A_{k_1}\}}{h}B_1$$

$$-\frac{h^{\alpha_2}\{\zeta_1 I_{k_1-1}-(\theta_1+\mu_1+\kappa_1)A_{k_1-1}\}}{h}A_{\alpha_1,k_1,2}),$$

$$R_{m+1}=R_0+\frac{(1-\alpha_1)}{ABC(\alpha_2)}\{\eta_1 D(t_m)+\theta_1 A(t_m)-(\nu_1+\xi_1)R(t_m)\}$$

$$+\frac{\alpha_2}{ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{h^{\alpha_1}\{\eta_1 D_{k_1}+\theta_1 A_{k_1}-(\nu_1+\xi_1)R_{k_1}\}}{h}B_1$$

$$-\frac{h^{\alpha_2}\{\eta_1 D_{k_1-1}+\theta_1 A_{k_1-1}-(\nu_1+\xi_1)R_{k_1-1}\}}{h}A_{\alpha_2,k_1,2}),$$

$$T_{m+1}=T_0+\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\mu_1 A(t_m)+\nu_1 R(t_m)-(\sigma_1+\tau_1)T(t_m)\}$$

$$+\frac{\alpha_1}{ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{h^{\alpha_2}\{\mu_1 A_{k_1}+\nu_1 R_{k_1}-(\sigma_1+\tau_1)T_{k_1}\}}{h}B_1$$

$$-\frac{h^{\alpha_1}\{\mu_1 A_{k_1-1}+\nu_1 R_{k_1-1}-(\sigma_1+\tau_1)T_{k_1-1}\}}{h}A_{\alpha_2,k_1,2}),$$

$$H_{m+1}=H_0+\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\lambda_1 I(t_m)+\rho_1 D(t_m)+\kappa_1 A(t_m)+\xi_1 R(t_m)+\sigma_1 T(t_m)\}$$

$$+\frac{\alpha_2}{ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{h^{\alpha_2}\{\lambda_1 I_{k_1}+\rho_1 D_{k_1}+\kappa_1 A_{k_1}+\xi_1 R_{k_1}+\sigma_1 T_{k_1}\}}{h}B_1$$

$$-\frac{h^{\alpha_2}\{\lambda_1 I_{k_1-1}+\rho_1 D_{k_1-1}+\kappa_1 A_{k_1-1}+\xi_1 R_{k_1-1}+\sigma_1 T_{k_1-1}\}}{h}A_{\alpha_2,k_1,2}),$$

$$E_{m+1}=E_0+\frac{(1-\alpha_2)}{ABC(\alpha_2)}\{\tau_1 T(t_m)\}$$

$$+\frac{\alpha_2}{ABC(\alpha_2)}\sum_{k_1=0}^{m}(\frac{h^{\alpha_1}\{\tau_1 T_{k_1}\}}{h}B_1$$

$$-\frac{h^{\alpha_2}\{\tau_1 T_{k_1-1}\}}{h}A_{\alpha_2,k_1,2}),$$

where

$$A_1=\{m+1-k_1)^{\alpha_2+1}-(m-k_1)^{\alpha_2}(m-k_1+1+\alpha_2)\}$$

and

$$A_2=\{m+1-k_1)^{\alpha_2}(m-k_1+2+\alpha_2)-(m-k_1)^{\alpha_2}(m-k_1+2+2\alpha_2)\}.$$

5. Results and Discussion

A Covid-19 fractional-order model for assessing the efficiency of coronavirus transmission in the community is presented in the paper. The model makes use of the sophisticated Atangana Toufik approach with Mittag-Leffler kernel, Sumudu transformations, and Atangana-Baleanu in the Caputo sense. Fractional values are used to detect nonlinear system memory outcomes and simulations show changes in the dynamics of the model.

In Figures 1, 5 and 6, $S(t)$, $R(t)$ and $T(t)$ start increasing by decreasing fractional values respectively while $H(t)$ and $E(t)$ in figure 7 and 8 start decreasing by decreasing fractional values after certain time respectively. In Figures 2, 3 and 4, $I(t)$, $D(t)$ and $A(t)$ started increasing and reached a peak, but after some time ultimate, all these compartments approach zero, which converge to steady-state. This shows that all infected compartments, either undetected or confirmed or ailing symptomatic infected, rise to a peak, but after a certain time, all individual comes to zero due to treatment and social distancing. It is also observed that susceptible and recovered individuals start increasing and approach a stable position. Also, deaths occur when infected are at peak position, as can be seen in figures 2, 3, 4 and 8.

As the fractional value falls, it is evident from all of the figures. The fractional-order derivatives, which have been shown to explain physical processes more accurately than the classical-order case, are the most important and reliable component. We found that outcomes at smaller fractional values are more accurate and converge to steady states faster than those at larger and integer values. In order to comprehend model behavior, the study also contrasted integer and fractional order. The results will help researchers in future investigations. The memory properties of the ABC derivative, which can't be achieved with integer-order derivatives, were also investigated in the study. Theoretical insights were represented graphically using Matlab. It has been discovered that fractional derivatives can more efficiently control COVID-19 in less time. Numerical results that have been provided illustrate the dynamics' behaviors in the different fractional orders.

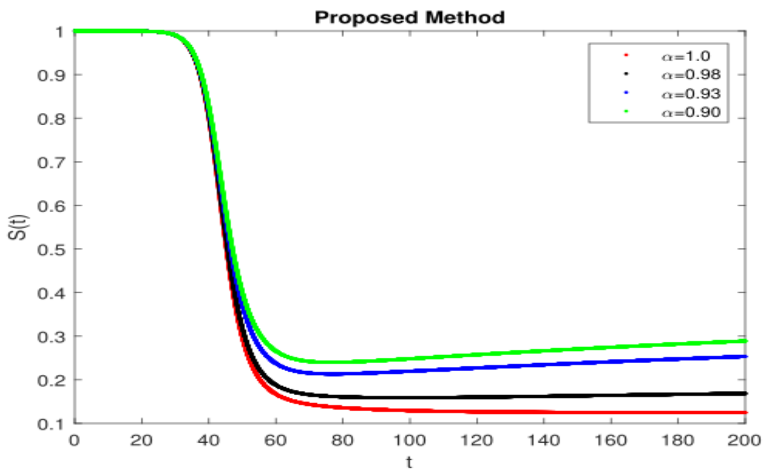

Figure 1: S compartment simulation dynamics via ABC derivative

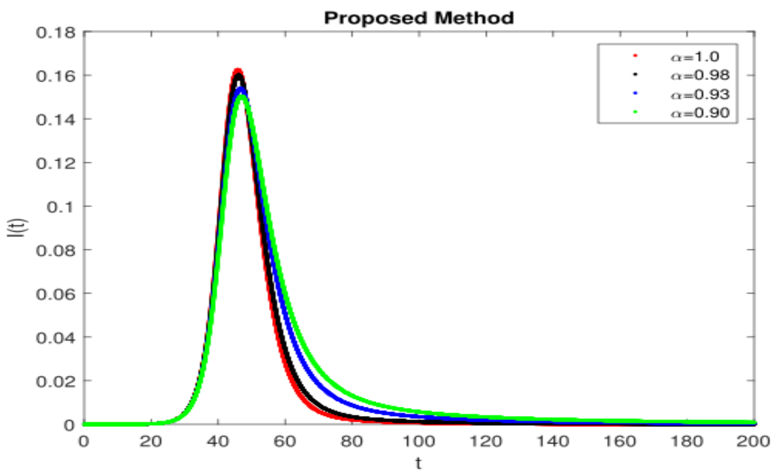

Figure 2: I compartment simulation dynamics via ABC derivative

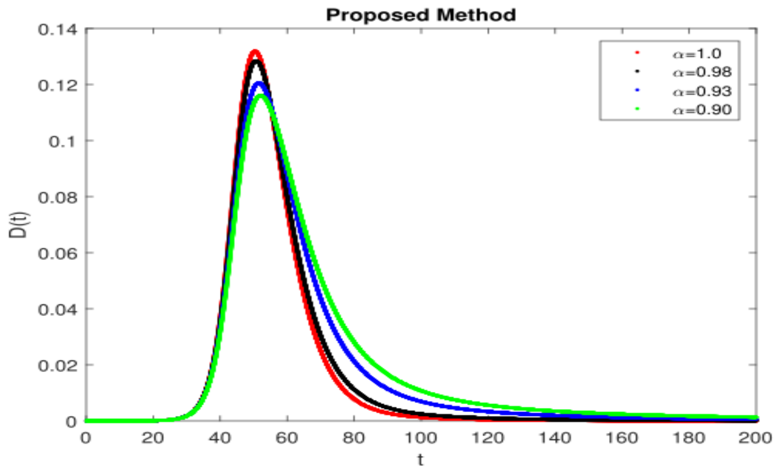

Figure 3: D compartment simulation dynamics via ABC derivative

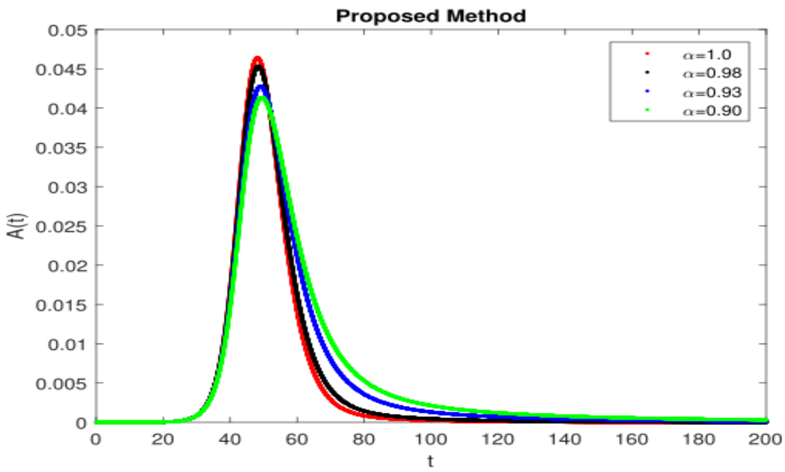

Figure 4: A compartment simulation dynamics via ABC derivative

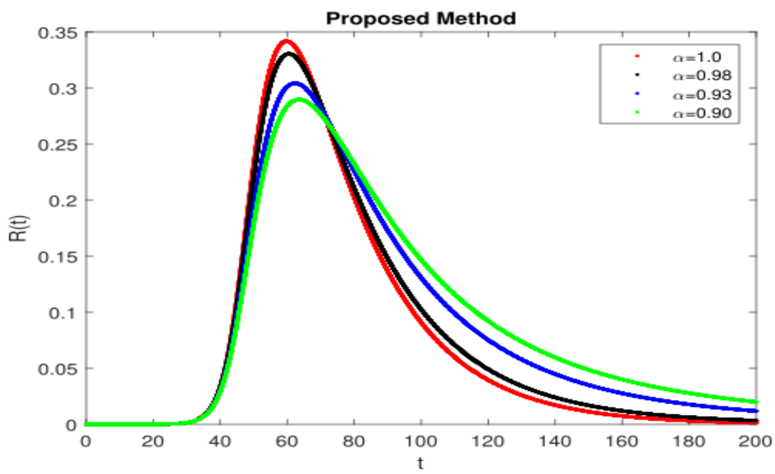

Figure 5: R compartment simulation dynamics via ABC derivative

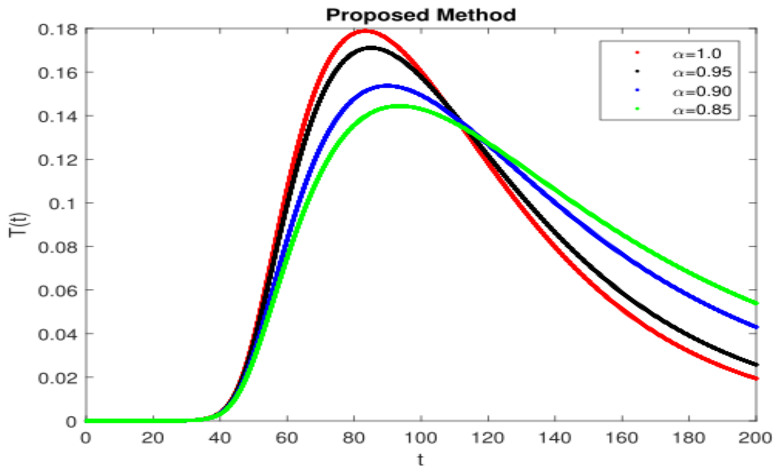

Figure 6: T compartment simulation dynamics via ABC derivative

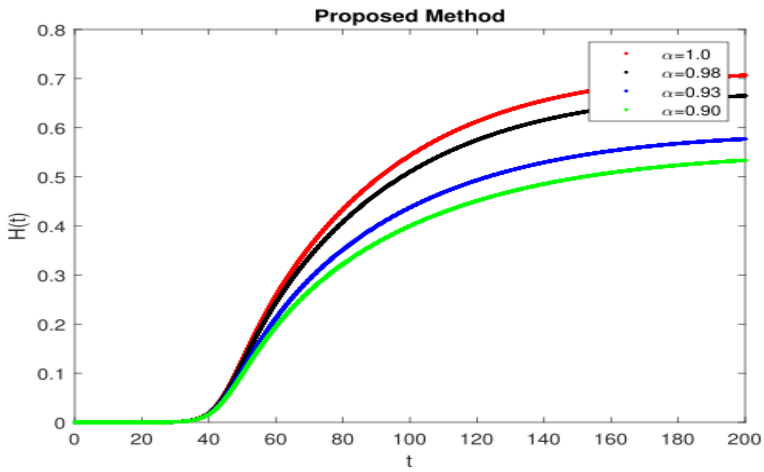

Figure 7: H compartment simulation dynamics via ABC derivative

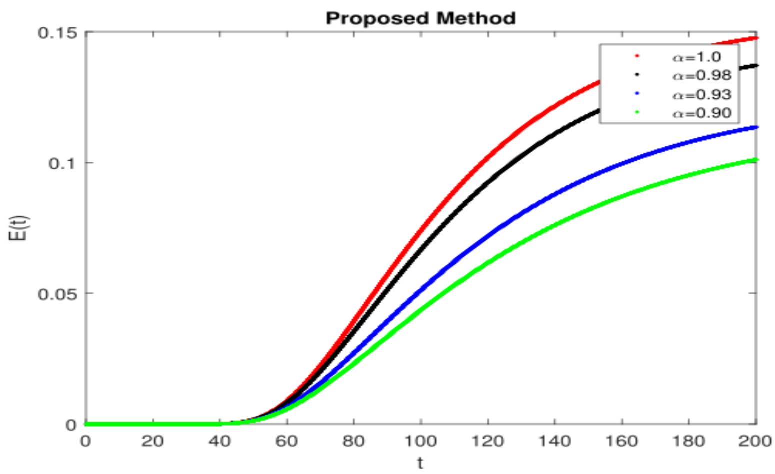

Figure 8: E compartment simulation dynamics via ABC derivative

6. Conclusion

This study uses a fractional operator to analyze the dynamic behavior of the COVID-19 model. Using Sumudu transforms and the Atangana Toufik scheme on fractional order differential equations, the model is developed via Atangana-Baleanu Caputo sense. Iterative techniques and fixed point theory are used to examine the model's uniqueness, stability, boundedness, and positivity. The Atangana-Baleanu Caputo sense and the Atangana Toufik with Mittag-Leffler kernel are used to take the arbitrary derivative of fractional order. A non-singular and non-local kernel is used to raise non-linear fractional differential equations from derivative. For efficiency verification, the outcomes are contrasted between integer-order and non-integer-order parameters. Simulations are conducted to confirm the actual behaviour of COVID-19 in society. The work highlights the importance of fractional parameters in handling the complexities of COVID-19 using numerical simulations, highlighting the need for comprehensive control methods to lower risk factors and understand viral transmission.

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