Journal of Mathematical Modeling and Fractional Calculus
https://acspublisher.com/journals/index.php/jmmfc
Received:08-08-2024, **Revised:** 10-10-2024,
Accepted:19-11-2024, **Published:** 31-12-2024,
https://doi.org/10.48165/jmmfc.2024.1206, 1(2),99-118.

An analysis of the Caputo Fabrizio fractional modeling for infectious diseases caused by chlamydia

Ali Hasan[a,*], Muhammad Azeem[a], Sana Ullah Saqib[b], Muhammad Manan Akram[c]

[a] Department of Mathematics and Statistics, The University of Lahore, Lahore 54590, Pakistan.
[b] Department of Applied Mathematics, National Chung Hsing University, Taichung, Taiwan.
[c] Washington university of Science and Technology ,USA.
Corresponding author Email: alihasanansari881@gmail.com

Abstract: These days, infectious illness mathematical modeling is a major global trend. With the use of current data, mathematical models enable us to predict the occurrence of disease outbreaks in the future. In this work, we use a fractal fractional operator with two fractal and fractional orders to solve a system of fractional differential equations using a Caputo Fabrizio type kernel. A six chambered model with a single source of chlamydia is studied using the concept of fractal fractional derivatives with nonsingular and nonlocal fading memory. The fractal fractional model of the Chlamydia system can be solved by using the characteristics of a non-decreasing and compact mapping. Initially, we calculate the system's equilibrium points and fundamental reproduction number R_0. We then look at the system's stability at the equilibrium point. Through the application of the Picard Lindelof methodology, we establish the existence of a unique solution for the given fractional CF-system of the hearing loss model and use fixed point theory to examine the stability of the iterative process. By taking into account the therapy as a control technique to lower the number of infected individuals, the system's optimal control is established. Calculating the estimated solution of the system involves applying the Euler technique for the fractional order Caputo Fabrizio derivative. In two scenarios, $R_0 < 1$ and $R_0 > 1$, we provide a numerical simulation of the disease's spread with regard to the basic reproduction number and the transmission rate. We compute the results for various fractional order derivatives and compare the findings in order to examine the impact of the fractional order derivative on the behavior and value of each variable in the model. Additionally, we examine the sensitivity of R_0 with regard to each model parameter and ascertain the influence of each parameter, taking into account the significance of reproduction number in the persistence of disease transmission. Finally, it can be said that once more, fractional operator mathematical models can help in making better decisions on how to manage financially turbulent situations.
Keywords: Chlamydia Model; Existence; Unicity and stability; Numerical scheme.

1. Introduction

The most common bacterial sexually transmitted infection in the UK is called Chlamydia trachomatis. The age groups with the highest prevalence of chlamydial infection include women between the ages of 16 and 19 and men between the ages of 20 and 24. Since most women with chlamydial infection are asymptomatic, some of them end up developing pelvic inflammatory disease as a result of not receiving treatment [1]. In 1999, there were approximately 92 million cases of chlamydia worldwide, comprising 42 million cases in men and 50 million cases in women [2]. Even though genuine resistance to Chlamydia trachomatis is uncommon, cases of recurrent chlamydia infections are still being reported after treatment with a single one g dose of azithromycin or a week's worth of doxycycline. This raises concerns regarding the possibility of azithromycin treatment failure. Even though reinfections account for the majority of repeat positive cases, new research suggests that treatment failure might also be a factor in [3]. A unique developmental cycle and obligatory intracellular lifestyle are shared by the evolutionarily distinct group of eubacteria known as the chlamydiae, which have been thoroughly studied in [4] under ideal cell culture conditions. We design and assess a Chlamydia trachomatis vaccination model with cost-effectiveness optimum control analysis. In [5] shows that the model's disease-free equilibrium is locally asymptotically stable when the reproduction number is less than unity. This allows researchers to examine the impact of various treatment combinations on the host dynamics of chronic genital chlamydial infections, which are characterized by the presence of IFN-γ-induced chlamydial persistence. Akinlotan et al. [6] create a mathematical framework of within host Chlamydia dynamics. A mathematical model that describes the kinetics of a Chlamydia trachomatis infection in a human carrier is described by Emuoyibofarhe et al. [7]. Relevant features like drug administration-assisted recovery were included in the model. The real solution looked at the model's solutions' existence and uniqueness. We analyze the model's stability both locally and globally. Researchers have provided a model to assess chlamydia therapeutic options in [8, 9, 10, 11], where they have presented some innovative studies on an epidemic model. Atangana [12] combined the two relevant fields of fractional and fractal calculus into a new class of concepts known as fractalfractional ideas. These operators have two components: the fractal dimension and the order. Differential equations with fractalfractional derivatives, according to [13, 14], convert the dimension and order of the putative system into a rational system. It takes fractional calculus to solve issues in the real world. It is frequently used in many different fields related to science, engineering, and finance. The characteristics of the fractional calculus that set it apart comprise fractional order derivatives and fractional integrals. The area of fractional calculus and its diverse aspects have garnered greater attention from scholars in recent times. This is because genetic mutations are an essential tool for understanding how various biological systems operate dynamically. The non-local characteristics of these component operators, that the integer separator operator [15, 16, 17, 18, 19, 20] lacks, give them their strength. The COVID-19 transmission pattern was examined in three severely affected nations in order to provide context for the modeling of COVID-19 transmission in [21]. For the time discretization and spatial discretization, respectively, [22, 23, 24, 25, 26, 27, 28] has used redefined extended B-spline functions and the Caputo time fractional derivative. In this work, we constructed several fractal fractional operators and employed various fractal differential operators [29, 30]. The duration of the fractional advection diffusion equation's approximate solution. Further information is provided in [31, 32, 33, 34].

A potential approach with several benefits is the use of fractalfractionaloperators with fractional and dual fractal orders in research studies. This method captures complicated patterns and abnormalities that other methods would miss, allowing for a more nuanced and detailed depiction of complex problems by simultaneously exploiting two orders. This improves our comprehension of complex processes and makes it easier to create more reliable mathematical models that are applicable to a wide range of academic fields. Using fractal fractional operators could transform a variety of industries, from image analysis to signal processing, by providing a flexible toolkit to tackle problems requiring a greater understanding of complex, multi-fractal phenomena. Accepting this novel paradigm opens up new avenues for research and discoveries by promoting a more exact and comprehensive approach in scientific studies. In this case, we analyze a fractalfractional model of contaminated lakes in terms of several distinct attributes. The following are the remaining sections of this study article: An introduction is given in section 1. The proposed framework and a number of fundamental fractional order derivatives that are useful in addressing the epidemiological framework are thoroughly presented in section 2. Section 3 provides a generalized version of the system and examines the qualitative features of the suggested model, such as existence and uniqueness and illness-free equilibrium. Section 4 examines the stability research of the proposed framework, including stability of Sumudu Transform. We examine the numerical scheme of the Caputo Fabrizio kernel in section 5. Sections 6 and 7 include the numerical simulations and findings, respectively.

2. Chlamydia viral mathematical model with Caputo Fabrizio

A deterministic compartmental system of the dynamics of Chlamydia illness transmission is presented. Researchers are looking into the causes and recurrence of potential epidemics. In order to understand viral transmission, let's examine the Vellappandi et al. [35] compartmental mathematical epidemic system and a few of its notable features. The following set of nonlinear ordinary differential equations can be used to illustrate the framework.

$$\begin{cases} {}^{CF}D_{0,t}^{\alpha,\beta}S(t) = (1-\phi)\Lambda - \frac{\beta S(I+\xi T)}{N} + wV - \mu S, \\ {}^{CF}D_{0,t}^{\alpha,\beta}V(t) = \phi\Lambda - (\mu + w)V - \frac{\beta(-\pi+1)V(\xi T+I)}{N}, \\ {}^{CF}D_{0,t}^{\alpha,\beta}E(t) = \frac{(-\pi+1)V\beta(\xi T+I)}{N} + \frac{\beta S(I+\xi T)}{N} - E(\sigma+\mu) + (-p+1)\theta T + \frac{\varepsilon\beta(I+\xi T)R}{N}, \\ {}^{CF}D_{0,t}^{\alpha,\beta}I(t) = \sigma E - (\gamma+\eta+\mu+\delta_1)I + p\theta T, \\ {}^{CF}D_{0,t}^{\alpha,\beta}T(t) = \eta I - (\mu+\theta+\delta_2+\tau)T, \\ {}^{CF}D_{0,t}^{\alpha,\beta}R(t) = \gamma I - \frac{\varepsilon\beta R(\xi T+I)}{N} + T\tau - \mu R. \end{cases} \tag{1}$$

where the starting circumstances are

$$S(0) = S^0 \geq 0, V(0) = V^0 \geq 0, T(0) = T^0 \geq 0, E(0) = E^0 \geq 0, I(0) = I^0 \geq 0, R(0) = R^0 \geq 0. \tag{2}$$

${}^{CF}D_{0,t}^{\alpha,\beta}$ represents the fractal fractional derivative with Caputo Fabrizio type kernel of fractional and fractal orders $\alpha \in (0,1]$ and $\beta \in (0,1]$, respectively. The proposed framework splits the entire population N into 6 parts at a particular time t. We developed the Chlamydia virus model's fractional order mathematical variant with vaccination in order to deal with the most Chlamydia model by using the most effective individuals and accounting for six (6) parts of populations. $S(t)$ symbolizes the people's unvaccinated susceptibility individuals over time t, $V(t)$ represents the population's vaccinated susceptibility individuals over time t, $E(t)$ represents the population's exposed individuals over time t, $I(t)$ represents the population's infected individuals over time t, $T(t)$ represents the population's treatment individuals over time t and $R(t)$ represents the population's recovered over time t. Table 1 contains the parameters defined for the suggested model.

Table 1: The parameters of the proposed system are discussed.

ϕ	Percentage of people who were recruited
Λ	Recruitment percentage
β	Rate of contact
ξ	Adjustment factor
N	Whole papulation
ω	Declining vaccination rate
μ	Spontaneous death rate
π	Vaccine's efficacy
σ	The speed with which those who are exposed spread infection
p	The percentage of patients not responding to treatment
θ	Treatment failure rate
ε	Rate of reinfection
γ	The speed at which the illness kills me
η	The speed at which I go toward T
δ_1	The speed at which the I kills me
δ_2	The speed at which the T kills me
τ	The speed at which T transitions to R

Thus, everyone in the population is ascertained via

$$N(t) = V(t) + T(t) + S(t) + R(t) + E(t) + I(t). \tag{3}$$

2.1. Fundamental Ideas of the Fractional Operator

Firstly, we jot down the following fundamentals of fractional calculus that will be useful in our investigation. This manuscript's current section reviews a few basic and auxiliary ideas regarding fractional operators.

Definition 2.1. *Let $n = [\alpha] + 1$ and assume that $\alpha \in (n-1, n)$. The fractional derivative of Caputo type for a function $\hat{F} \in AC_R^{(n)}([0,\infty))$ is represented by*

$$^{C}D_0^{\alpha,\beta}\hat{F}(t) = \int_0^t \frac{(t-\mathbb{C})^{n-\alpha-1}}{\Gamma(n-\alpha)}\hat{F}(\mathbb{C})d\mathbb{C}, \tag{4}$$

if the integral has a finite value [36, 43].

Definition 2.2. *Subsequently, two Italian mathematicians, Caputo and Fabrizio, create a new fractional operator with no singular kernel [38]. They presuppose that $\hat{F}(t) \in H^1(a,b)$ and $a < b$. Next, for a function $\hat{F}(t)$, the Caputo Fabrizio or (CF) derivative of an arbitrary order $\alpha \in (0,1)$ is given by*

$$^{CF}D_a^{\alpha}\hat{F}(t) = \frac{(2-\alpha)M(\alpha}{2(1-\alpha)}\int_a^t exp\left(\frac{-\alpha(t-\mathbb{C})}{1-\alpha}\right)\hat{F}'(\mathbb{C})d\mathbb{C}, \tag{5}$$

where $M(\alpha)$ is a normalization function depending on order α and $M(0) = M(1) = 1$.

Moreover, $^{CF}D_a^{(\alpha+n)}\hat{F}(t) =^{CF} D_a^{(\alpha)}(D^n\hat{F}(t))$ holds for $n \geq 1$ and $\alpha \in (0,1)$ in [39]. A new explicit formula for the function $M(\alpha)$ was discovered in 2015 by Losada and Nieto [40] as $M(\alpha) = \frac{2}{2-\alpha}$, where $\alpha \in (0,1)$. The fractional CF derivative for $\hat{F}(t)$ in this instance is represented by

$$^{CF}D_0^{\alpha}\hat{F}(t) = \frac{(1}{1-\alpha}\int_0^t exp\left(\frac{-\alpha(t-\mathbb{C})}{1-\alpha}\right)\hat{F}'(\mathbb{C})d\mathbb{C}. \tag{6}$$

It is evident that the equality $^{CF}D_0^{\alpha}\hat{F}(t)$ is identical to $\hat{F}(t) = A^*$, where A^* is an arbitrary constant, for all $\alpha \in (0,1)$.

Definition 2.3. *Furthermore, the fractional CF-integral of order $\alpha \in (0,1)$ for $\hat{F}(t)$ was defined by Losada and Nieto as follows*

$$^{CF}J_0^{(\alpha}\hat{F}(t) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\hat{F}(t) + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \hat{F}(\mathbb{C})d\mathbb{C}, \tag{7}$$

for any $t > 0$ [40].

Definition 2.4. *To achieve this, build the ensuing set by*

$$A = \{\hat{F}(t) : \exists\aleph,\ c_1, c_2 \geq 0\ sothat|\hat{F}(t)| < \aleph exp\left(\frac{t}{c_i}\right)\}, \tag{8}$$

where $t \in (-1)^i \times [0,\infty)$. Next, the Sumudu transform of a function $\hat{F}(t) \in A$ is defined as follows and is represented as $\mathbb{ST}[\hat{F}(t)](s) = \hat{F}(s)$.

$$\mathbb{ST}[\hat{F}(t)](s) = \hat{F}(s) = \frac{1}{s}\int_0^{\infty} exp\frac{t}{s}\hat{F}(\mathbb{C})d\mathbb{C}, \tag{9}$$

where $s \in (-c_1, c_2)$ for all $t \geq 0$ and the inverse Sumudu transform of $\hat{F}(s)$ is denoted by $\hat{F}(t) = \mathbb{ST}^{-1}[\hat{F}(s)]$. Let us now suppose that $\hat{F}(t)$ is a function, given the existence of its CF-derivative of fractional order. The fractional CF-derivative for $\hat{F}(t)$'s Sumudu transform is defined as

$$\mathbb{ST}[^{CF}D_0^{(\alpha)}\hat{F}(t)](s) = \frac{M(\alpha)}{1-\alpha+\alpha s}\left(\mathbb{ST}[\hat{F}(t)]s - \hat{F}(0)\right), \tag{10}$$

for all $t > 0$ [41].

3. Qualitative Evaluation of the Suggested Framework

We are currently conducting a qualitative analysis of our model of infectious disease with vaccination, which is expressed as a frame work of nonlinear differential equations (1), to better understand its features and the variables that govern the dynamics of infectious disease transmission.

3.1. Existence and Uniqueness of Suggested Model Solutions

Next, we use the Caputo-Fabrizio derivative to examine the following fractional model of Chlamydia infectious. We start by demonstrating that our framework (1) has a solution. To that, we apply fixed point theory. Let us first define the Banach space $M = W^6$, where $W = w(J,R)$, in order to perform our qualitative analysis.

$$\begin{cases} {}^{CF}D_{0,t}^{\alpha,\beta} S(t) = (1-\phi)\Lambda - \frac{\beta S(I+\xi T)}{N} + wV - \mu S, \\ {}^{CF}D_{0,t}^{\alpha,\beta} V(t) = \phi\Lambda - (\mu + w)V - \frac{\beta(-\pi+1)V(\xi T+I)}{N}, \\ {}^{CF}D_{0,t}^{\alpha,\beta} E(t) = \frac{(-\pi+1)V\beta(\xi T+I)}{N} + \frac{\beta S(I+\xi T)}{N} - E(\sigma+\mu) + (-p+1)\theta T + \frac{\varepsilon\beta(I+\xi T)R}{N}, \\ {}^{CF}D_{0,t}^{\alpha,\beta} I(t) = \sigma E - (\gamma+\eta+\mu+\delta_1)I + p\theta T, \\ {}^{CF}D_{0,t}^{\alpha,\beta} T(t) = \eta I - (\mu+\theta+\delta_2+\tau)T, \\ {}^{CF}D_{0,t}^{\alpha,\beta} R(t) = \gamma I - \frac{\varepsilon\beta R(\xi T+I)}{N} + T\tau - \mu R. \end{cases} \tag{11}$$

We apply the Picard Lindelof technique to determine whether solutions exist for the modified fractional system (11) of the Chlamydia infectious model. The Chlamydia infectious model (11) must first be transformed into a fractional integral equation in order to accomplish this.

$$\begin{aligned} S(t) = S_0 &+ \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{(1-\phi)\Lambda - \frac{\beta S(I+\xi T)}{N} + wV - \mu S\right\} \\ &+ \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \left\{(1-\phi)\Lambda - \frac{\beta S(\mathbb{C})(I(\mathbb{C})+\xi T(\mathbb{C}))}{N} + wV(\mathbb{C}) - \mu S(\mathbb{C})\right\} d\mathbb{C}, \end{aligned} \tag{12}$$

$$\begin{aligned} V(t) = V_0 &+ \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\phi\Lambda - (\mu+w)V - \frac{\beta(-\pi+1)V(\xi T+I)}{N}\right\} \\ &+ \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \left\{\phi\Lambda - (\mu+w)V(\mathbb{C}) - \frac{\beta(-\pi+1)V(\mathbb{C})(\xi T(\mathbb{C})+I(\mathbb{C}))}{N}\right\} d\mathbb{C}, \end{aligned} \tag{13}$$

$$\begin{aligned} E(t) = E_0 &+ \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\frac{(-\pi+1)V\beta(\xi T+I)}{N} + \frac{\beta S(I+\xi T)}{N} - E(\sigma+\mu) + (-p+1)\theta T\right. \\ &\left. + \frac{\varepsilon\beta(I+\xi T)R}{N}\right\} + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \left\{\frac{(-\pi+1)V(\mathbb{C})\beta(\xi T(\mathbb{C})+I(\mathbb{C}))}{N}\right. \\ &\left. + \frac{\beta S(\mathbb{C})(I(\mathbb{C})+\xi T(\mathbb{C}))}{N} - E(\mathbb{C})(\sigma+\mu) + (-p+1)\theta T(\mathbb{C}) + \frac{\varepsilon\beta(I(\mathbb{C})+\xi T(\mathbb{C}))R}{N}\right\} d\mathbb{C}, \end{aligned} \tag{14}$$

$$\begin{aligned} I(t) = I_0 &+ \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\{\sigma E - (\gamma+\eta+\mu+\delta_1)I + p\theta T\} \\ &+ \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \{\sigma E(\mathbb{C}) - (\gamma+\eta+\mu+\delta_1)I(\mathbb{C}) + p\theta T(\mathbb{C})\} d\mathbb{C}, \end{aligned} \tag{15}$$

$$\begin{aligned} T(t) = T_0 &+ \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\{\eta I - (\mu+\theta+\delta_2+\tau)T\} \\ &+ \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \{\eta I(\mathbb{C}) - (\mu+\theta+\delta_2+\tau)T(\mathbb{C})\} d\mathbb{C}, \end{aligned} \tag{16}$$

$$R(t) = R_0 + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\gamma I - \frac{\varepsilon\beta R(\xi T + I)}{N} + T\tau - \mu R\right\} + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \left\{\gamma I(\mathbb{C}) - \frac{\varepsilon\beta R(\mathbb{C})(\xi T(\mathbb{C}) + I(\mathbb{C}))}{N} + T(\mathbb{C})\tau - \mu R(\mathbb{C})\right\} d\mathbb{C}. \tag{17}$$

Now, we define the Picard iterative method as follows ($m = 0, 1, 2, ...$), paying careful attention to equations (12)-(17).

$$S_{m+1}(t) = S_0 + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{(1-\phi)\Lambda - \frac{\beta S_m(I_m + \xi T_m)}{N} + wV_m - \mu S_m\right\} + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \left\{(1-\phi)\Lambda - \frac{\beta S_m(\mathbb{C})(I_m(\mathbb{C}) + \xi T_m(\mathbb{C}))}{N} + wV_m(\mathbb{C}) - \mu S_m(\mathbb{C})\right\} d\mathbb{C}, \tag{18}$$

$$V(t) = V_0 + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\phi\Lambda - (\mu + w)V_m - \frac{\beta(-\pi+1)V_m(\xi T_m + I_m)}{N}\right\} + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \left\{\phi\Lambda - (\mu + w)V_m(\mathbb{C}) - \frac{\beta(-\pi+1)V_m(\mathbb{C})(\xi T_m(\mathbb{C}) + I_m(\mathbb{C}))}{N}\right\} d\mathbb{C}, \tag{19}$$

$$\begin{aligned} E(t) = E_0 &+ \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\frac{(-\pi+1)V_m\beta(\xi T_m + I_m)}{N} + \frac{\beta S_m(I_m + \xi T_m)}{N} - E_m(\sigma+\mu) + (-p+1)\theta T_m \right.\\ &\left. + \frac{\varepsilon\beta(I_m + \xi T_m)R_m}{N}\right\} + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \left\{\frac{(-\pi+1)V_m(\mathbb{C})\beta(\xi T_m(\mathbb{C}) + I_m(\mathbb{C}))}{N}\right.\\ &\left. + \frac{\beta S_m(\mathbb{C})(I_m(\mathbb{C}) + \xi T_m(\mathbb{C}))}{N} - E_m(\mathbb{C})(\sigma+\mu) + (-p+1)\theta T_m(\mathbb{C}) + \frac{\varepsilon\beta(I_m(\mathbb{C}) + \xi T_m(\mathbb{C}))R_m}{N}\right\} d\mathbb{C}, \end{aligned} \tag{20}$$

$$I(t) = I_0 + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\{\sigma E_m - (\gamma + \eta + \mu + \delta_1) I_m + p\theta T_m\} + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \{\sigma E_m(\mathbb{C}) - (\gamma + \eta + \mu + \delta_1) I_m(\mathbb{C}) + p\theta T_m(\mathbb{C})\} d\mathbb{C}, \tag{21}$$

$$T(t) = T_0 + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\{\eta I_m - (\mu + \theta + \delta_2 + \tau) T_m\} + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \{\eta I_m(\mathbb{C}) - (\mu + \theta + \delta_2 + \tau) T_m(\mathbb{C})\} d\mathbb{C}, \tag{22}$$

$$R(t) = R_0 + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\gamma I_m - \frac{\varepsilon\beta R_m(\xi T_m + I_m)}{N} + T_m\tau - \mu R_m\right\} + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \left\{\gamma I_m(\mathbb{C}) - \frac{\varepsilon\beta R_m(\mathbb{C})(\xi T_m(\mathbb{C}) + I_m(\mathbb{C}))}{N} + T_m(\mathbb{C})\tau - \mu R_m(\mathbb{C})\right\} d\mathbb{C}. \tag{23}$$

We now assume that anytime n goes to infinity, the exact solutions of the fractional system (11) can be obtained by taking the limit from both sides of (18)-(23). Put differently, the following is how the solutions are obtained

$$\begin{cases} \lim_{m\to\infty} S_m = S(t), \\ \lim_{m\to\infty} V_m = V(t), \\ \lim_{m\to\infty} E_m = E(t), \\ \lim_{m\to\infty} I_m = I(t), \\ \lim_{m\to\infty} T_m = T(t), \\ \lim_{m\to\infty} R_m = R(t). \end{cases} \tag{24}$$

We may now apply the Picard-Lindelof method to determine an existence criteria and the uniqueness of the solutions. In order to accomplish this, define the subsequent operators

$$\begin{cases} M_1(t,S(t)) = (1-\phi)\Lambda - \frac{\beta S(I+\xi T)}{N} + wV - \mu S, \\ M_2(t,V(t)) = \phi\Lambda - (\mu+w)V - \frac{\beta(-\pi+1)V(\xi T+I)}{N}, \\ M_3(t,E(t)) = \frac{(-\pi+1)V\beta(\xi T+I)}{N} + \frac{\beta S(I+\xi T)}{N} - E(\sigma+\mu) + (-p+1)\theta T + \frac{\varepsilon\beta(I+\xi T)R}{N}, \\ M_4(t,S(t)) = \sigma E - (\gamma+\eta+\mu+\delta_1)I + p\theta T, \\ M_5(t,T(t)) = \eta I - (\mu+\theta+\delta_2+\tau)T, \\ M_6(t,R(t)) = \gamma I - \frac{\varepsilon\beta R(\xi T+I)}{N} + T\tau - \mu R. \end{cases} \tag{25}$$

Where, for the first, second, third, forth, fifth and sixth functions respectively, $M_1(t,S(t))$, $M_2(t,V(t))$, $M_3(t,E(t))$, $M_4(t,I(t))$, $M_5(t,T(t))$ and $M_6(t,R(t))$ are regarded as contractions with respect to $S(t)$, $V(t)$, $E(t)$, $I(t)$, $T(t)$ and $R(t)$. We also take into consideration the following product spaces

$$\begin{cases} N_{l,q_1}(t,S(t)) = [t-l,t+l]\times[S-q_1,S+q_1] = L\times Q_1, \\ N_{l,q_2}(t,V(t)) = [t-l,t+l]\times[V-q_1,V+q_1] = L\times Q_2, \\ N_{l,q_3}(t,E(t)) = [t-l,t+l]\times[E-q_1,E+q_1] = L\times Q_3, \\ N_{l,q_4}(t,I(t)) = [t-l,t+l]\times[I-q_1,I+q_1] = L\times Q_4, \\ N_{l,q_5}(t,T(t)) = [t-l,t+l]\times[T-q_1,T+q_1] = L\times Q_5, \\ N_{l,q_6}(t,R(t)) = [t-l,t+l]\times[R-q_1,R+q_1] = L\times Q_6. \end{cases} \tag{26}$$

Consider the following

$$M_1^* = \sup_{(t,S)\in N_{l,q_1}} \|M_1(t,S(t))\|, \quad M_2^* = \sup_{(t,V)\in N_{l,q_2}} \|M_2(t,V(t))\|, \quad M_3^* = \sup_{(t,E)\in N_{l,q_3}} \|M_3(t,E(t))\|,$$
$$M_4^* = \sup_{(t,I)\in N_{l,q_4}} \|M_4(t,I(t))\|, \quad M_5^* = \sup_{(t,T)\in N_{l,q_5}} \|M_5(t,T(t))\|, \quad M_6^* = \sup_{(t,R)\in N_{l,q_6}} \|M_6(t,R(t))\|.$$

The Picard operator is defined at the current position

$$\aleph : C(L,Q_1,Q_2,Q_3,Q_4,Q_5,Q_6) \to C(L,Q_1,Q_2,Q_3,Q_4,Q_5,Q_6),$$

via means of

$$\aleph(\hat{F}(t)) = \hat{F}_0(t) + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\mathscr{H}(t,\hat{F}(t)) + \frac{2\alpha}{(2-\alpha)M(\hat{\alpha})}\int_0^t \mathscr{H}(\mathbb{C},\hat{F}(\mathbb{C}))d\mathbb{C}, \tag{27}$$

provided that $\hat{F}(t) = \{S(t),V(t),E(t),I(t),T(t),R(t)\}$ and $\hat{F}_0(t) = \{S_0,V_0,E_0,I_0,T_0,R_0\}$ and

$$\mathscr{H}(t,\hat{F}(t)) = \{M_1(t,S(t)),M_2(t,V(t)),M_3(t,E(t)),M_4(t,I(t)),M_5(t,T(t)),M_6(t,R(t))\}. \tag{28}$$

We define a uniform norm as follows on the space $C(L,Q_1,Q_2,Q_3,Q_4,Q_5,Q_6)$ in order to use the Picard theorem. A $\|\hat{F}(t)\|_\infty = \sup_{t\in[t-l,t+l]}|\hat{F}(t)|$. Let's assume for the purposes of this that all solution functions are constrained over a certain time span.

$$\|\hat{F}(t)\|_\infty \le \max\{q_1,q_2,q_3,q_4,q_5,q_6\} = q. \tag{29}$$

In addition, suppose that $M^* = \max\{M_1^*, M_2^*, M_3^*, M_4^*, M_5^*, M_6^*\}$ and that t_0 is a constant, where $t \leq t_0$. Next, we have

$$
\begin{aligned}
&\| \aleph(\hat{F}(t)) - \hat{F}_0(t)\|, \\
&= \| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \mathscr{H}(t, \hat{F}(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \mathscr{H}(\mathbb{C}, \hat{F}(\mathbb{C})) d\mathbb{C} \|, \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \|\mathscr{H}(t, \hat{F}(t))\| + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \|\mathscr{H}(\mathbb{C}, \hat{F}(\mathbb{C}))\| d\mathbb{C}, \\
&\leq \left\{ \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} + \frac{2\alpha t_0}{(2-\alpha)M(\alpha)} \right\} M^*, \\
&\leq \check{U} M^*, \\
&\leq q,
\end{aligned}
$$

where $\check{U} = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} + \frac{2\alpha t_0}{(2-\alpha)M(\alpha)}$ and $\check{U} < \frac{q}{M^*}$ are assumed. Lastly, our goal is to establish the contraction nature of the Picard operator $\aleph$. Since the functions M_1, M_2, M_3, M_4, M_5 and M_6 are contractions, we can write for each $\hat{F}_1(t), \hat{F}_2(t) \in C(L, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6)$ in order to accomplish this purpose.

$$
\|\mathscr{H}(t, \hat{F}_1(t)) - \mathscr{H}(t, \hat{F}_2(t))\| \leq Y^* \|\hat{F}_1(t) - \hat{F}_2(t)\|, \tag{30}
$$

where the contraction constant is $Y^* < 1$. Right now, utilizing the definition of the Picard operator $\aleph$ found in (27), by inequality (30), and by the equality

$$
\| \aleph(\hat{F}_1(t)) - \aleph(\hat{F}_2(t))\| = \sup_{t \in [t-l, t+l]} |\hat{F}_1(t) - \hat{F}_2(t)|,
$$

$$
\begin{aligned}
\| \aleph(\hat{F}_1(t)) - \aleph(\hat{F}_2(t))\| &= \bigg\| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \left[\mathscr{H}(t, \hat{F}_1(t)) - \mathscr{H}(t, \hat{F}_2(t)) \right] \\
&\quad + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \left[\mathscr{H}(\mathbb{C}, \hat{F}_1(\mathbb{C})) - \mathscr{H}(\mathbb{C}, \hat{F}_2(\mathbb{C})) \right] d\mathbb{C} \bigg\|, \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \left\| \mathscr{H}(t, \hat{F}_1(t)) - \mathscr{H}(t, \hat{F}_2(t)) \right\| \\
&\quad + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \left\| \mathscr{H}(\mathbb{C}, \hat{F}_1(\mathbb{C})) - \mathscr{H}(\mathbb{C}, \hat{F}_2(\mathbb{C})) \right\| d\mathbb{C}, \\
&\leq \frac{2(1-\alpha)Y^*}{(2-\alpha)M(\alpha)} \left\| \hat{F}_1(t) - \hat{F}_2(t) \right\| + \frac{2\alpha Y^*}{(2-\alpha)M(\alpha)} \int_0^t \left\| \hat{F}_1(\mathbb{C}) - \hat{F}_2(\mathbb{C}) \right\| d\mathbb{C}, \\
&\leq \left[\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} + \frac{2\alpha}{(2-\alpha)M(\alpha)} \right] Y^* \left\| \hat{F}_1(t) - \hat{F}_2(t) \right\|, \\
&\leq \check{U} Y^* \left\| \hat{F}_1(t) - \hat{F}_2(t) \right\|,
\end{aligned}
$$

As a result, we get

$$
\| \aleph(\hat{F}_1(t)) - \aleph(\hat{F}_2(t))\| \leq \check{U} Y^* \left\| \hat{F}_1(t) - \hat{F}_2(t) \right\|.
$$

This shows that since $Y^* < 1$, the operator $\aleph$ is a contraction with constant $\check{U} Y^* < 1$. Therefore, the Chlamydia infectious model's fractional system (17) appears to have a unique solution, according to the Banach fixed point theorem.

3.2. Positivity and Restriction of Solutions

Given that the proposed model's responses are bounded and guaranteed to be positive, the study looks at the conditions under which it can be applied to significant value real world scenarios. Regarding classical derivatives, we possess the subsequent information $\forall t \geq 0$.

$$\begin{cases} S(t) \geq S_0 e^{-(\beta|I|_\infty+\xi|T|_\infty+\mu)t}, \\ V(t) \geq V_0 e^{-(\mu+\frac{(1-\pi)(\xi T+I)\beta}{N})+\omega}, \\ E(t) \geq E_0 e^{-(\sigma+\mu)}, \\ I(t) \geq I_0 e^{-(\gamma+\eta+\mu+\delta_1)}, \\ T(t) \geq T_0 e^{-(\mu+\theta+\delta_2+\tau)}, \\ R(t) \geq R_0 e^{-(\mu+\frac{\varepsilon\beta(\xi T+I)}{N})}. \end{cases} \tag{31}$$

We must determine the norm

$$\|\mathfrak{W}\|_\infty = \sup_{t\in D_{\mathfrak{W}}} |\mathfrak{W}(t)|. \tag{32}$$

We discover

$$\begin{aligned} {}^{CF}\mathrm{D}_t^\alpha S(t) &= \Lambda(-\phi+1) - \frac{S\beta(I+T\xi)}{N} + \omega V - \mu S, \\ &\geq -S(\frac{(\beta(I+T\xi)}{N} + \mu), \\ &\geq -S(\frac{\beta(|I|+|T|\xi)}{N} + \mu), \\ &\geq -(\mu + \beta(\sup_{t\in D_I}|I| + \xi \sup_{t\in T}|T|))S, \\ &\geq -(\mu + \beta|I|_\infty + |T|_\infty\xi)S. \end{aligned} \tag{33}$$

For the classical derivative, the outcome is $\forall t \geq 0$.

$$S(t) \geq S_0 e^{-(\beta|I|_\infty+\xi|T|_\infty+\mu)t}. \tag{34}$$

While the next section discusses positive options with non-local operators, states that the system (1)'s solutions will undoubtedly remain positive when all of the beginning criteria for nonlocal operator are satisfied. For $\forall t > 0$, we build a fractal-fractional operator with a Mittag Leffler kernel.

$$\begin{cases} S(t) \geq S(0)E_\alpha\left[-\frac{\upsilon^{1-\beta}\alpha(|I|_\infty\beta+|T|_\infty\xi+\mu)t^\alpha}{\tilde{AB}(\alpha)-(1-\alpha)(|I|_\infty\beta+|T|_\infty\xi+\mu)}\right], \\ V(t) \geq V(0)E_\alpha\left[-\frac{\upsilon^{1-\beta}\alpha(\mu+\omega+\frac{\beta(1-\pi)(\xi|T|_\infty+|I|_\infty)}{N})t^\alpha}{\tilde{AB}(\alpha)-(1-\alpha)(\omega+\mu+\frac{(-\pi+1)\beta(|T|_\infty\xi+|I|_\infty)}{N})}\right], \\ E(t) \geq E(0)E_\alpha\left[-\frac{\upsilon^{1-\beta}\alpha(\sigma+\mu)t^\alpha}{\tilde{AB}(\alpha)-(1-\alpha)(\sigma+\mu)}\right], \\ I(t) \geq I(0)E_\alpha\left[-\frac{\upsilon^{1-\beta}\alpha(\gamma+\eta+\mu+\delta_1)t^\alpha}{\tilde{AB}(\alpha)-(1-\alpha)(\gamma+\eta+\mu+\delta_1)}\right], \\ T(t) \geq T(0)E_\alpha\left[-\frac{\upsilon^{1-\beta}\alpha(\mu+\theta+\delta_2+\tau)t^\alpha}{\tilde{AB}(\alpha)-(1-\alpha)(\mu+\theta+\delta_2+\tau)}\right], \\ R(t) \geq R(0)E_\alpha\left[-\frac{\upsilon^{1-\beta}\alpha(\mu+\frac{\varepsilon\beta(\xi|T|_\infty+|I|_{infty})}{N})t^\alpha}{\tilde{AB}(\alpha)-(1-\alpha)(\mu+\frac{\varepsilon\beta(\xi|T|_\infty+|I|_\infty)}{N})}\right]. \end{cases} \tag{35}$$

where the time component is υ.

Lemma 3.1. *The region* $\Upsilon_l \in \mathbb{R}_+^6$

$$\Upsilon_l = \{(V(t), E(t), S(t), I(t), R(t), T(t)) \in \mathbb{R}_+^6\} \tag{36}$$

attracts all of system (1)'s solutions and, for the suggested system in $\mathbb{R}_+^6$*, is positively invariant when utilized with initial constraints that are not negative.*

Proof. We will illustrate the framework (1) advantageous outcome, and the results are:

$$\begin{cases} {}^{CF}D_{0,t}^{\alpha,\beta}S(t)\Big|_{S=0} = (1-\phi)\Lambda + wV \geq 0, \\ {}^{CF}D_{0,t}^{\alpha,\beta}V(t)\Big|_{V=0} = \phi\Lambda \geq 0, \\ {}^{CF}D_{0,t}^{\alpha,\beta}E(t)\Big|_{E=0} = \frac{(-\pi+1)V\beta(\xi T+I)}{N} + \frac{\beta S(I+\xi T)}{N} + (-p+1)\theta T + \frac{\varepsilon\beta(I+\xi T)R}{N} \geq 0, \\ {}^{CF}D_{0,t}^{\alpha,\beta}I(t)\Big|_{I=0} = \sigma E + p\theta T \geq 0, \\ {}^{CF}D_{0,t}^{\alpha,\beta}T(t)\Big|_{T=0} = \eta I \geq 0, \\ {}^{CF}D_{0,t}^{\alpha,\beta}R(t)\Big|_{R=0} = \gamma I + T\tau \geq 0. \end{cases} \tag{37}$$

In accordance with system (37), the vector field is situated in the area $\mathbb{R}_+^6$ around every hyperplane encircling the non-negative orthant about $t > 0$. Therefore,

$$\Upsilon_l = \{(S(t),V(t),I(t),T(t),E(t),R(t)) \in \mathbb{R}_+^6\}, \tag{38}$$

is a positively invariant domain. □

3.3. Equilibrium points analysis

This section offers a thorough examination of equilibrium points. First, we solve the framework 1 for equilibrium points. The areas devoid of illness are

$$E_1\left(S^*,V^*,T^*,I^*,R^*,E^*\right) = \left(\frac{(\mu+w)(1-\phi)\Lambda + \phi\Lambda w}{(w+\mu)\mu}, \frac{\Lambda\phi}{\mu+w}, 0,0,0,0\right).$$

The EEPs (Endemic Equilibrium Points) at this time are

$$E_1\left(S^*,V^*,E^*,I^*,T^*,R^*\right).$$

where

$$S^{**}(t) = \frac{(-\phi+1)\Lambda + \omega V^{**}}{(\lambda^{**}+\mu)},$$

$$V^{**}(t) = \frac{\phi\Lambda}{((1-\pi)\lambda^{**}+\mu+\omega)},$$

$$T^{**}(t) = \frac{\eta I^{**}}{K_3},$$

$$E^{**}(t) = \frac{[S^{**}+(-\pi+1)V^{**}]\lambda^{**} + (-p+1)\varphi T^{**} + \varepsilon\lambda^{**}R^{**}}{K_1},$$

$$\lambda^{**} = \frac{\beta\left(I^{**}+T^{**}\xi\right)}{N^{**}},$$

$$I^{**}(t) = \frac{K_3\sigma\lambda^{**}\left(\mu+\varepsilon\lambda^{**}\right)\left[S^{**}+(1-\pi)V^{**}\right]}{K_4},$$

$$R^{**}(t) = \frac{\left(K_3\gamma+\eta\tau\right)I^{**}}{K_3\left(\mu+\varepsilon\lambda^{**}\right)}.$$

where, $P_1 = \sigma+\mu, P_2 = \eta+\mu+\delta_1+\gamma, P_3 = \varphi+\mu+\tau+\delta_2$.

$$\begin{aligned} P_4 = \big[&\mu^2\left(\gamma+\mu+\delta_1\right)\left(\delta_2+\mu+\tau\right) + \sigma\mu\left(\gamma+\mu+\delta_1\right)\left(\varphi+\mu+\tau+\delta_2\right) + \mu\sigma\eta\left(\mu+\delta_2+\tau\right) \\ &+ \varepsilon\lambda^{**}\mu\left(\gamma+\mu+\delta_1\right)\left(\delta_2+\mu+\tau\right) + \varepsilon\lambda^{**}\sigma\left(\mu+\delta_1\right)\left(\mu+\varphi+\delta_2+\tau\right) + \varepsilon\lambda^{**}\sigma\eta\left(\mu+\delta_2\right) \\ &+ \varepsilon\lambda^{**}\mu\eta\varphi(1-p) + \mu^2\varphi\eta(1-p)\big]. \end{aligned}$$

3.4. Reproduction Number

Taking the maximum eigen value of the spectral radius FV^{-1} yields the basic reproduction number $\mathscr{R}_0$.

$$\mathscr{R}_o = \frac{\sigma\left(\eta\xi\beta A + \beta A G_4\right)}{\left[G_4\mu\left(\gamma+\mu+\delta_1\right)+\left(\delta_2+\mu+\tau\right)\left(\sigma\eta+\eta\mu\right)+\eta\mu\varphi(-p+1)\right]N}, \tag{39}$$

where, $A = (S^* + (-\pi+1)V^*), J_1 = \omega+\mu, J_2 = \sigma+\mu, J_3 = \eta+\mu+\delta_1+\gamma, J_4 = \varphi+\mu+\tau+\delta_2$, $N = S^*+T^*+E^*+I^*+V^*+R^*$.

4. Stability Analysis of the Proposed Model

Using the Sumudu transform, we offer an iterative technique for analyzing the stability of the fractional hearing loss CF framework (11). To achieve this goal, we receive

$$\begin{cases}
\mathbb{ST}\left\{{}^{CF}D_0^\alpha S(t)\right\}(s) = \mathbb{ST}\left\{(1-\phi)\Lambda - \frac{\beta S(I+\xi T)}{N} + wV - \mu S\right\}(s),\\
\mathbb{ST}\left\{{}^{CF}D_0^\alpha V(t)\right\}(s) = \mathbb{ST}\left\{\phi\Lambda - (\mu+w)V - \frac{\beta(-\pi+1)V(\xi T+I)}{N}\right\}(s),\\
\mathbb{ST}\left\{{}^{CF}D_0^\alpha E(t)\right\}(s) = \mathbb{ST}\left\{\frac{(-\pi+1)V\beta(\xi T+I)}{N} + \frac{\beta S(I+\xi T)}{N}\right.\\
\left. -E(\sigma+\mu) + (-p+1)\theta T + \frac{\varepsilon\beta(I+\xi T)R}{N}\right\}(s),\\
\mathbb{ST}\left\{{}^{CF}D_0^\alpha I(t)\right\}(s) = \mathbb{ST}\left\{\sigma E - (\gamma+\eta+\mu+\delta_1)I + p\theta T\right\}(s),\\
\mathbb{ST}\left\{{}^{CF}D_0^\alpha T(t)\right\}(s) = \mathbb{ST}\left\{\eta I - (\mu+\theta+\delta_2+\tau)T\right\}(s),\\
\mathbb{ST}\left\{{}^{CF}D_0^\alpha R(t)\right\}(s) = \mathbb{ST}\left\{\gamma I - \frac{\varepsilon\beta R(\xi T+I)}{N} + T\tau - \mu R\right\}(s).
\end{cases} \tag{40}$$

Considering the specification of the Sumudu transform for the fractional CF-derivative with appropriate attention, we obtain

$$\begin{cases}
\frac{M(\alpha}{1-\alpha+\alpha s}\left\{\mathbb{ST}[S(t)](s) - S(0)\right\} = \mathbb{ST}\left\{(1-\phi)\Lambda - \frac{\beta S(I+\xi T)}{N} + wV - \mu S\right\}(s),\\
\frac{M(\alpha}{1-\alpha+\alpha s}\left\{\mathbb{ST}[V(t)](s) - V(0)\right\} = \mathbb{ST}\left\{\phi\Lambda - (\mu+w)V - \frac{\beta(-\pi+1)V(\xi T+I)}{N}\right\}(s),\\
\frac{M(\alpha)}{1-\alpha+\alpha s}\left\{\mathbb{ST}[E(t)](s) - E(0)\right\} = \mathbb{ST}\left\{\frac{(-\pi+1)V\beta(\xi T+I)}{N} + \frac{\beta S(I+\xi T)}{N}\right.\\
\left. -E(\sigma+\mu) + (-p+1)\theta T + \frac{\varepsilon\beta(I+\xi T)R}{N}\right\}(s),\\
\frac{M(\alpha)}{1-\alpha+\alpha s}\left\{\mathbb{ST}[I(t)](s) - I(0)\right\} = \mathbb{ST}\left\{\sigma E - (\gamma+\eta+\mu+\delta_1)I + p\theta T\right\}(s),\\
\frac{M(\alpha)}{1-\alpha+\alpha s}\left\{\mathbb{ST}[T(t)](s) - T(0)\right\} = \mathbb{ST}\left\{\eta I - (\mu+\theta+\delta_2+\tau)T\right\}(s),\\
\frac{M(\alpha)}{1-\alpha+\alpha s}\left\{\mathbb{ST}[R(t)](s) - R(0)\right\} = \mathbb{ST}\left\{\gamma I - \frac{\varepsilon\beta R(\xi T+I)}{N} + T\tau - \mu R\right\}(s).
\end{cases} \tag{41}$$

The following equalities can be obtained by rewriting the formulas

$$\begin{cases}
\mathbb{ST}[S(t)](s) = S(0) + \frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{(1-\phi)\Lambda - \frac{\beta S(I+\xi T)}{N} + wV - \mu S\right\}(s),\\
\mathbb{ST}[V(t)](s) = V(0) + \frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\phi\Lambda - (\mu+w)V - \frac{\beta(-\pi+1)V(\xi T+I)}{N}\right\}(s),\\
\mathbb{ST}[E(t)](s) = E(0) + \frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\frac{(-\pi+1)V\beta(\xi T+I)}{N} + \frac{\beta S(I+\xi T)}{N}\right.\\
\left. -E(\sigma+\mu) + (-p+1)\theta T + \frac{\varepsilon\beta(I+\xi T)R}{N}\right\}(s),\\
\mathbb{ST}[I(t)](s) = I(0) + \frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\sigma E - (\gamma+\eta+\mu+\delta_1)I + p\theta T\right\}(s),\\
\mathbb{ST}[T(t)](s) = T(0) + \frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\eta I - (\mu+\theta+\delta_2+\tau)T\right\}(s),\\
\mathbb{ST}[R(t)](s) = R(0) + \frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\gamma I - \frac{\varepsilon\beta R(\xi T+I)}{N} + T\tau - \mu R\right\}(s).
\end{cases} \tag{42}$$

Following the inverse Sumudu transform on both sides of the system (43), the fractional CF-model (11) recursive equations are now produced

$$
\begin{cases}
S_{m+1}(t) = S_m(0) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{(1-\phi)\Lambda - \frac{\beta S_m(I_m+\xi T_m)}{N} + wV - \mu S_m\right\}(s)\right], \\
V_{m+1}(t) = V_m(0) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\phi\Lambda - (\mu+w)V_m - \frac{\beta(-\pi+1)V_m(\xi T_m+I_m)}{N}\right\}(s)\right], \\
E_{m+1}(t) = E_m(0) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\frac{(-\pi+1)V_m\beta(\xi T_m+I_m)}{N} + \frac{\beta S_m(I_m+\xi T_m)}{N}\right.\right. \\
\left.\left. -E_m(\sigma+\mu) + (-p+1)\theta T_m + \frac{\varepsilon\beta(I_m+\xi T_m)R_m}{N}\right\}(s)\right], \\
I_{m+1}(t) = I_m(0) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\sigma E_m - (\gamma+\eta+\mu+\delta_1)I_m + p\theta T_m\right\}(s)\right], \\
T_{m+1}(t) = T_m(0) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\eta I_m - (\mu+\theta+\delta_2+\tau)T_m\right\}(s)\right], \\
R_{m+1}(t) = R_m(0) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\gamma I_m - \frac{\varepsilon\beta R_m(\xi T_m+I_m)}{N} + T_m\tau - \mu R_m\right\}(s)\right].
\end{cases} \tag{43}
$$

Conversely, the aforementioned CF-system's approximations are produced by

$$
S(t) = \lim_{m\to\infty} S_m(t), \quad V(t) = \lim_{m\to\infty} V_m(t), \quad E(t) = \lim_{m\to\infty} E_m(t),
$$
$$
I(t) = \lim_{m\to\infty} I_m(t), \quad T(t) = \lim_{m\to\infty} T_m(t), \quad R(t) = \lim_{m\to\infty} R_m(t).
$$

Now, taking into account the aforementioned concepts and relationships, we can verify the stability of the fractional CF-system.

Theorem 4.1. *Assume that Υ has the following definition for a self-map*

$$
\begin{cases}
\Upsilon(S_m(t)) = S_{m+1}(t) = S_m(t) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{(1-\phi)\Lambda - \frac{\beta S_m(I_m+\xi T_m)}{N} + wV - \mu S_m\right\}(s)\right], \\
\Upsilon(V_m(t)) = V_{m+1}(t) = V_m(t) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\phi\Lambda - (\mu+w)V_m - \frac{\beta(-\pi+1)V_m(\xi T_m+I_m)}{N}\right\}(s)\right], \\
\Upsilon(E_m(t)) = E_{m+1}(t) = E_m(t) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\frac{(-\pi+1)V_m\beta(\xi T_m+I_m)}{N} + \frac{\beta S_m(I_m+\xi T_m)}{N}\right.\right. \\
\left.\left. -E_m(\sigma+\mu) + (-p+1)\theta T_m + \frac{\varepsilon\beta(I_m+\xi T_m)R_m}{N}\right\}(s)\right], \\
\Upsilon(I_m(t)) = I_{m+1}(t) = I_m(t) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\sigma E_m - (\gamma+\eta+\mu+\delta_1)I_m + p\theta T_m\right\}(s)\right], \\
\Upsilon(T_m(t)) = T_{m+1}(t) = T_m(t) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\eta I_m - (\mu+\theta+\delta_2+\tau)T_m\right\}(s)\right], \\
\Upsilon(R_m(t)) = R_{m+1}(t) = R_m(t) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{\gamma I_m - \frac{\varepsilon\beta R_m(\xi T_m+I_m)}{N} + T_m\tau - \mu R_m\right\}(s)\right].
\end{cases} \tag{44}
$$

Then, whenever we have, the iterative fractional CF-system (43) is Υ stable in $L^1(a,b)$.

$$
\begin{cases}
1 + w\Psi_1 + \mu\Psi_2 + \beta\frac{\Theta_1^*\Psi_3+\Theta_2^*\Psi_4+\xi\{\Theta_3^*\Psi_5+\Theta_2^*\Psi_6\}}{N}, \\
1 + (\mu+w)\Psi_7 + \beta(-\pi+1)\frac{\Theta_1^*\Psi_8+\Theta_2^*\Psi_9+\xi\{\Theta_3^*\Psi_{10}+\Theta_2^*\Psi_{11}\}}{N}, \\
1 + \beta(-\pi+1)\frac{\Theta_1^*\Psi_{12}+\Theta_2^*\Psi_{13}+\xi\{\Theta_3^*\Psi_{14}+\Theta_2^*\Psi_{15}\}}{N} \\
+\beta\frac{\Theta_1^*\Psi_{16}+\Theta_2^*\Psi_{17}+\xi\{\Theta_3^*\Psi_{18}+\Theta_2^*\Psi_{19}\}}{N} + (\sigma+\mu)\Psi_{20} + (-p+1)\theta\Psi_{21} \\
+\beta\varepsilon\frac{\Theta_1^*\Psi_{22}+\Theta_{12}^*\Psi_{23}+\xi\{\Theta_3^*\Psi_{24}+\Theta_{12}^*\Psi_{25}\}}{N}, \\
1 + \sigma\Psi_{26} + (\gamma+\eta+\mu+\delta_1)\Psi_{27} + p\theta\Psi_{28}, \\
1 + \eta\Psi_{29} + (\mu+\theta+\tau+\delta_2)\Psi_{30}, \\
1 + \gamma\Psi_{31} + \tau\Psi_{32} + \mu\Psi_{33} + \beta\varepsilon\frac{\Theta_1^*\Psi_{34}+\Theta_{12}^*\Psi_{35}+\xi\{\Theta_3^*\Psi_{36}+\Theta_{12}^*\Psi_{37}\}}{N}.
\end{cases} \tag{45}
$$

where functions Ψ_i for $i = 1, 2, \cdots, 8$ are added in the sequel.

Proof. Our goal in starting the proof is to demonstrate that operator Υ has a fixed point. Each time $u, v \in N$, we could write

$$
\|\Upsilon(S_u(t)) - \Upsilon(S_v(t))\| = \|S_u(t) - S_v(t)\|,
$$

$$= \Big\| S_u(t) + \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{(1-\phi)\Lambda - \frac{\beta S_u(I_u+\xi T_u)}{N} + wV_u - \mu S_u\right\}(s)\right]$$
$$- S_v(t) - \mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\left\{(1-\phi)\Lambda - \frac{\beta S_v(I_v+\xi T_v)}{N} + wV_v - \mu S_v\right\}(s)\right]\Big\|,$$
$$\leq \Big\|S_u(t) - S_v(t)\Big\| + \Big\|\mathbb{ST}^{-1}\Big[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\Big\{-\beta\frac{S_u(I_u+\xi T_u) - S_v(I_v+\xi T_v)}{N}$$
$$+ w(V_u - V_v) - \mu(S_u - S_v)\Big\}(s)\Big]\Big\|,$$
$$\leq \Big\|S_u(t) - S_v(t)\Big\| + \Big\|\mathbb{ST}^{-1}\Big[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\Big\{-\beta\frac{S_uI_u + \xi S_uT_u - S_vI_v - \xi S_vT_v)}{N}$$
$$+ w(V_u - V_v) - \mu(S_u - S_v)\Big\}(s)\Big]\Big\|,$$

$$\begin{aligned} &\leq \Big\|S_u(t) - S_v(t)\Big\| + \Big\|\mathbb{ST}^{-1}\Big[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\Big\{w(V_u - V_v) - \mu(S_u - S_v) \\ &- \beta\frac{I_u(S_u - S_v) + S_v(I_u - I_v) + \xi\left\{T_u(S_u - S_v) + S_v(T_u - T_v)\right\}}{N}\Big\}(s)\Big]\Big\|. \end{aligned} \tag{46}$$

Due to the fact that all four solutions play the same role, we will think about

$$\|S_u - S_v\| \simeq \|I_u - I_v\| \simeq \|V_u - V_v\| \simeq \|T_u - T_v\|. \tag{47}$$

Next, we have (46) and (47)

$$\begin{aligned} &\leq \Big\|S_u(t) - S_v(t)\Big\| + \Big\|\mathbb{ST}^{-1}\Big[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\Big\{w(S_u - S_v) - \mu(S_u - S_v) \\ &- \beta\frac{I_u(S_u - S_v) + S_v(S_u - S_v) + \xi\left\{T_u(S_u - S_v) + S_v(S_u - S_v)\right\}}{N}\Big\}(s)\Big]\Big\|, \end{aligned}$$

I_u, S_v and T_u are bounded sequences since they are convergent. Thus, for every t and every $u, v \in N$, we get constants Θ_1^*, Θ_2^* and Θ_3^*. Then

$$\begin{aligned} &\|I_u\| \leq \Theta_1^*, \quad \|S_v\| \leq \Theta_2^*, \quad \|T_u\| \leq \Theta_3^*, \quad \|T_v\| \leq \Theta_4^*, \quad \|S_u\| \leq \Theta_5^*, \quad \|I_v\| \leq \Theta_6^*, \\ &\|E_u\| \leq \Theta_7^*, \quad \|E_v\| \leq \Theta_8^*, \quad \|V_u\| \leq \Theta_9^*, \quad \|V_v\| \leq \Theta_{10}^*, \quad \|R_u\| \leq \Theta_{11}^*, \quad \|R_v\| \leq \Theta_{12}^*. \end{aligned}$$

As a result, we acquire

$$\begin{aligned} &\leq \Big\|S_u(t) - S_v(t)\Big\| + \mathbb{ST}^{-1}\Big[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}\Big\{w\|(S_u - S_v)\| + \mu\|(S_u - S_v)\| \\ &+ \beta\frac{\Theta_1^*\|(S_u - S_v)\| + \Theta_2^*\|(S_u - S_v)\| + \xi\left\{\Theta_3^*\|(S_u - S_v)\| + \Theta_2^*\|(S_u - S_v)\|\right\}}{N}\Big\}(s)\Big], \end{aligned}$$

$$\|\Upsilon(S_u(t)) - \Upsilon(S_v(t))\| \leq \left\{1 + w\Psi_1 + \mu\Psi_2 + \beta\frac{\Theta_1^*\Psi_3 + \Theta_2^*\Psi_4 + \xi\left\{\Theta_3^*\Psi_5 + \Theta_2^*\Psi_6\right\}}{N}\right\}\|(S_u - S_v)\|,$$

where Ψ_j, for $j = 1, 2, 3, \cdots, 6$ are functions coming from $\mathbb{ST}^{-1}\left[\frac{1-\alpha+\alpha s}{M(\alpha)}\mathbb{ST}[.]\right]$. The similar way, we obtain

$$\|\Upsilon(V_u(t)) - \Upsilon(V_v(t))\| \leq \left\{1 + (\mu + w)\Psi_7 + \beta(-\pi + 1)\frac{\Theta_1^*\Psi_8 + \Theta_2^*\Psi_9 + \xi\left\{\Theta_3^*\Psi_{10} + \Theta_2^*\Psi_{11}\right\}}{N}\right\}\|(V_u - V_v)\|,$$

$$\|\Upsilon(E_u(t))-\Upsilon(E_v(t))\| \leq \Big\{1+\beta(-\pi+1)\frac{\Theta_1^*\Psi_{12}+\Theta_2^*\Psi_{13}+\xi\left\{\Theta_3^*\Psi_{14}+\Theta_2^*\Psi_{15}\right\}}{N}$$
$$+\beta\frac{\Theta_1^*\Psi_{16}+\Theta_2^*\Psi_{17}+\xi\left\{\Theta_3^*\Psi_{18}+\Theta_2^*\Psi_{19}\right\}}{N}+(\sigma+\mu)\Psi_{20}+(-p+1)\theta\Psi_{21}$$
$$+\beta\varepsilon\frac{\Theta_1^*\Psi_{22}+\Theta_{12}^*\Psi_{23}+\xi\left\{\Theta_3^*\Psi_{24}+\Theta_{12}^*\Psi_{25}\right\}}{N}\Big\}\|(E_u-E_v)\|,$$
$$\|\Upsilon(I_u(t))-\Upsilon(I_v(t))\| \leq \{1+\sigma\Psi_{26}+(\gamma+\eta+\mu+\delta_1)\Psi_{27}+p\theta\Psi_{28}\}\|(I_u-I_v)\|,$$
$$\|\Upsilon(T_u(t))-\Upsilon(T_v(t))\| \leq \{1+\eta\Psi_{29}+(\mu+\theta+\tau+\delta_2)\Psi_{30}\}\|(T_u-T_v)\|,$$

and

$$\|\Upsilon(R_u(t))-\Upsilon(R_v(t))\| \leq \Big\{1+\gamma\Psi_{31}+\tau\Psi_{32}+\mu\Psi_{33}$$
$$+\beta\varepsilon\frac{\Theta_1^*\Psi_{34}+\Theta_{12}^*\Psi_{35}+\xi\left\{\Theta_3^*\Psi_{36}+\Theta_{12}^*\Psi_{37}\right\}}{N}\Big\}\|(R_u-R_v)\|.$$

The self-map Υ has a fixed point because it is contraction, according to the assumptions (45). We assert that Υ satisfies every assumption of theorem. This claim can be easily proven by assuming that $\Theta=(0,0,0,0,0,0)$ additionally

$$\Theta=\begin{cases} 1+w\Psi_1+\mu\Psi_2+\beta\frac{\Theta_1^*\Psi_3+\Theta_2^*\Psi_4+\xi\left\{\Theta_3^*\Psi_5+\Theta_2^*\Psi_6\right\}}{N}, \\ 1+(\mu+w)\Psi_7+\beta(-\pi+1)\frac{\Theta_1^*\Psi_8+\Theta_2^*\Psi_9+\xi\left\{\Theta_3^*\Psi_{10}+\Theta_2^*\Psi_{11}\right\}}{N}, \\ 1+\beta(-\pi+1)\frac{\Theta_1^*\Psi_{12}+\Theta_2^*\Psi_{13}+\xi\left\{\Theta_3^*\Psi_{14}+\Theta_2^*\Psi_{15}\right\}}{N} \\ +\beta\frac{\Theta_1^*\Psi_{16}+\Theta_2^*\Psi_{17}+\xi\left\{\Theta_3^*\Psi_{18}+\Theta_2^*\Psi_{19}\right\}}{N}+(\sigma+\mu)\Psi_{20}+(-p+1)\theta\Psi_{21} \\ +\beta\varepsilon\frac{\Theta_1^*\Psi_{22}+\Theta_{12}^*\Psi_{23}+\xi\left\{\Theta_3^*\Psi_{24}+\Theta_{12}^*\Psi_{25}\right\}}{N}, \\ 1+\sigma\Psi_{26}+(\gamma+\eta+\mu+\delta_1)\Psi_{27}+p\theta\Psi_{28}, \\ 1+\eta\Psi_{29}+(\mu+\theta+\tau+\delta_2)\Psi_{30}, \\ 1+\gamma\Psi_{31}+\tau\Psi_{32}+\mu\Psi_{33}+\beta\varepsilon\frac{\Theta_1^*\Psi_{34}+\Theta_{12}^*\Psi_{35}+\xi\left\{\Theta_3^*\Psi_{36}+\Theta_{12}^*\Psi_{37}\right\}}{N}. \end{cases}$$

After that, theorem presumptions are all met, making Υ Picard Υ stable, and the proof is ultimately finished. □

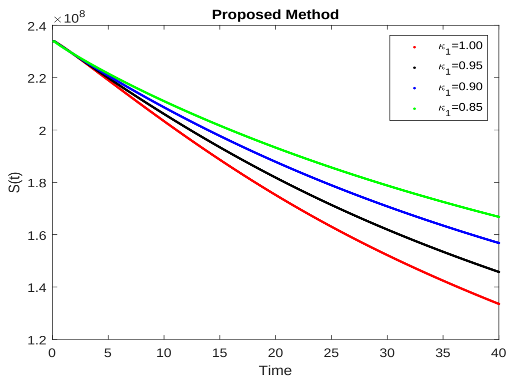
5. Numerical Scheme with generalized form of fractal-fractional Operator

Now, to derive the numerical solution of the given Caputo-Fabrizio type model (1), we use the Newton polynomial method. Using the related polynomial equations of the proposed model classes, $\hat{\alpha}\in[0,1]$, $0\leq t\leq T$, step size $h=T/N$, and $t_n=nh$, for $n=0,1,2,\ldots,N\in Z^+$, the solution of the model (1) is written as follows

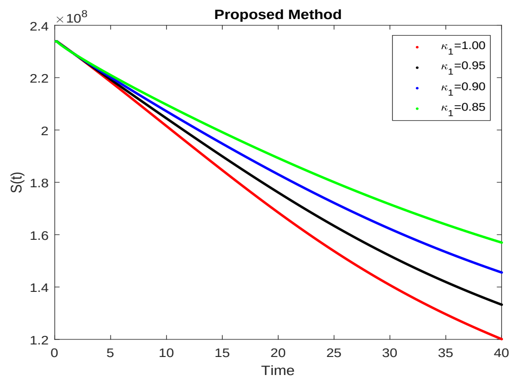
$$\begin{aligned} Y^{n+1} &= Y^n+\frac{1-\alpha}{M(\alpha)}\left[t_n^{1-\alpha}g(t_n,y(t_n))-t_{n-1}^{1-\alpha}g(t_{n-1},y(t_{n-1}))\right] \\ &+\frac{\alpha}{M(\alpha)}\left[\frac{23}{12}t_n^{1-\alpha}g(t_n,y(t_n))\Delta t-\frac{4}{3}t_{n-1}^{1-\alpha}g(t_{n-1},y(t_{n-1}))\Delta t+\frac{5}{12}t_{n-2}^{1-\alpha}g(t_{n-2},y(t_{n-2}))\Delta t\right]. \end{aligned} \tag{48}$$

6. Result and Discussion

A numerical simulation was conducted to analyze the control of newly designed complicated fractional chaotic sexually transmitted disease chlamydia in United states using the fractal fractional approach. $S(0)=233824096$, $T(0)=250000$, $E(0)=500000$, $V(0)=1000000$, $I(0)=200904$ and $R(0)=50000$ are the initial conditions of the proposed system. The variables $S(t),T(t),I(t),V(t),E(t)$, and $R(t)$, respectively, indicate susceptible individuals, treated human, infectious peoples, vaccinated group, exposed population and humans with recovery. Using the proposed system's fractal fractional derivative, we can easily see that, according to data from different nations, more accurate estimate of the minimum illness rate values from [35] is given by vulnerable persons, vaccinated susceptible individuals, exposed peoples, infectious peoples, treated humans, and humans with recovery rate. Figures (1-6) illustrate this impact at various fractal and fractional order values at different dimension. Using the CF fractional derivatives, figures 1 simulates $S(t)$ shows a noticeable decrease in the susceptible population under the influence of control measures, figures 2 simulates $V(t)$ shows a noticeable decrease in the vaccinated susceptible population under the effectiveness of the implemented control is evident, figures 3 shows a noticeable decrease in the exposed peoples increase by fractional orders under the influence of control measures, figures 4 simulates $I(t)$ shows a noticeable increase in the infected population under the effectiveness of the implemented control is evident, figures 5 simulates $T(t)$ shows a noticeable decrease in the treated humans population under the effectiveness of the implemented control is evident and figures 6 simulates $T(t)$ shows a noticeable increase in the recovered humans population under the effectiveness of the implemented control is evident at different fractional orders with different dimension. This study sheds light on how disease control is expected to develop in the future and how we may improve our efforts to reduce the spread of infectious disease in society. Compared to classical derivatives, fractal fractional analysis produces robust results for all compartments when examining steady states with non-integer order derivatives. It's also important to remember that fractional value reduction improves the accuracy and reliability of solutions for every compartment. Memory influence can be observed with a greater degree of freedom and a solution restricted to a steady state point that lies in a feasible range by varying the fractal dimension. The fractional order results are more dependable than the integer order ones due to the fractional order models' ability to capture memory effects in a system, despite the fact that both models' predictive strengths are rather equal.

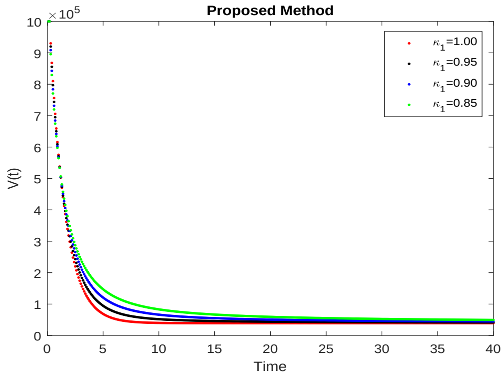


(a) At $\alpha = 0.9$

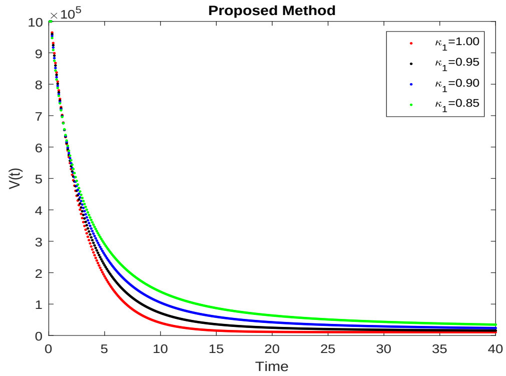


(b) At $\alpha = 1$

Figure 1: $S(t)$ simulation using parametric values of dimension 1.

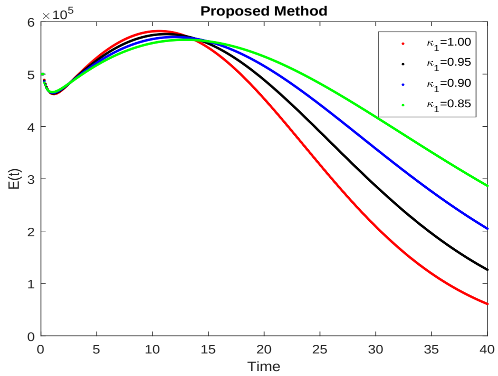


(a) At $\alpha = 0.9$

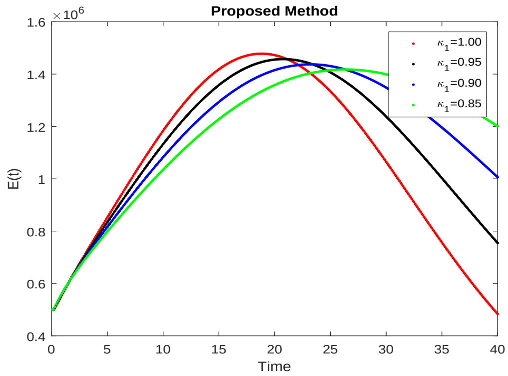


(b) At $\alpha = 1$

Figure 2: $V(t)$ simulation using parametric values of dimension 1.

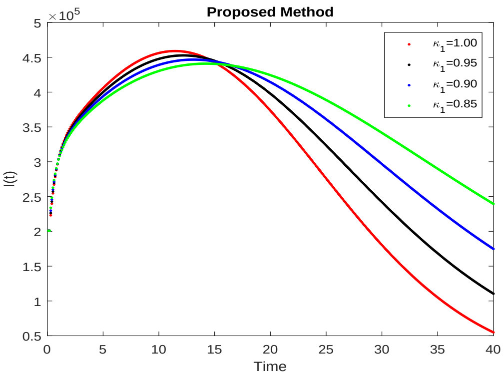


(a) At $\alpha = 0.9$

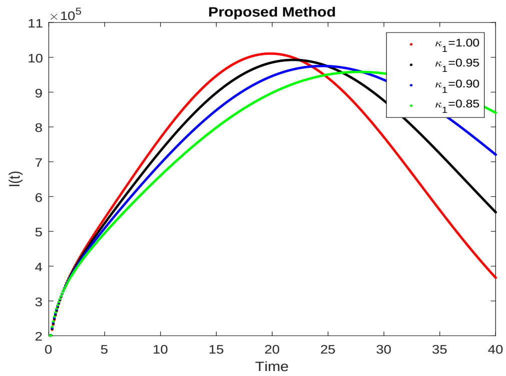


(b) At $\alpha = 1$

Figure 3: $E(t)$ simulation using parametric values of dimension 1.

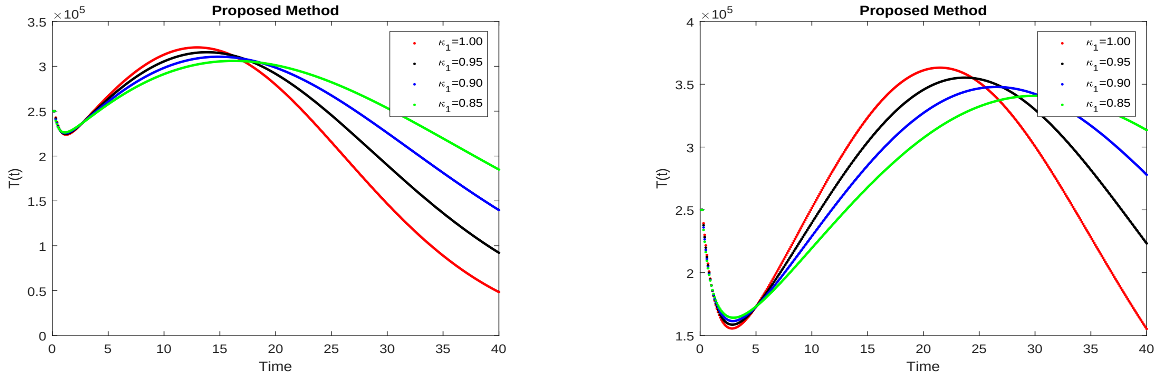


(a) At $\alpha = 0.9$



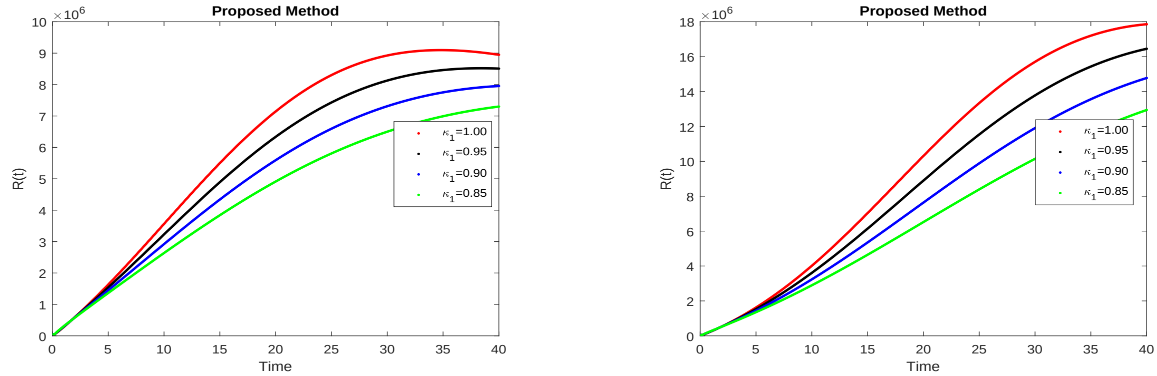
(b) At $\alpha = 1$

Figure 4: $I(t)$ simulation using parametric values of dimension 1.



(a) At $\alpha = 0.9$ **(b)** At $\alpha = 1$

Figure 5: $T(t)$ simulation using parametric values of dimension 1.



(a) At $\alpha = 0.9$ **(b)** At $\alpha = 1$

Figure 6: $R(t)$ simulation using parametric values of dimension 1.

7. Conclusions

This research investigates the transmission of Chlamydia in the United States with varying rates using a dynamic, chaotic fractional order model with a fractal fractional derivative. Non-singular and nonlocal kernel, which emerge in the derivation of the generalized fractal operator, provide the foundation of this fractional model. The uniqueness and positivity of the solutions that fall within the feasible zone were satisfied by the model. Stable and Chlamydia system behavior in conceivable locations are the results of the chaos control requirements being satisfied. The Chlamydia model is examined theoretically and numerically using a fractal fractional operator understanding of the CF function at various fractal and fractional order values. The results are also compared using exponential decay kernel at various fractional order values, with the proportion of minimum interest rates in various countries used as a proxy. The fractional-order Chlamydia model, which has been adjusted with a fractal fractional derivative, is used to regulate the essential lowest infectious rate of the disease, indicating strong consensus on the system's Chlamydia disease management. Based on numerical data, the model gives an effect analysis of the essential minimum infectious rate. The graph depicts the influence of variables on the quantity of the critical minimum infectious rate over time. This research approach has significant results for disease coefficients, rate of infection, and demand for recovery. As vaccination demand and infection exponents begin to drop, infectious rates begin to rise in accordance with the initial conditions, exposing the Chlamydia system's true macroeconomic behavior. It is highlighted here that, for non-integer time-fractional parameters, when compared to time-integer parameters, the intricate chaotic fractional structure yields more reliable and suitable results.

Funding: None

Conflict of interest: None

References

[1] Manavi, K. (2006). A review on infection with Chlamydia trachomatis. Best Practice & Research Clinical Obstetrics & Gynaecology, 20(6), 941-951.

[2] Hillis, S. D., & Wasserheit, J. N. (1996). Screening for chlamydiaa key to the prevention of pelvic inflammatory disease. New England Journal of Medicine, 334(21), 1399-1401.

[3] Kong, F. Y. S., & Hocking, J. S. (2015). Treatment challenges for urogenital and anorectal Chlamydia trachomatis. BMC infectious diseases, 15, 1-7.

[4] Hogan, R. J., Mathews, S. A., Mukhopadhyay, S., Summersgill, J. T., & Timms, P. (2004). Chlamydial persistence: beyond the biphasic paradigm. Infection and immunity, 72(4), 1843-1855.

[5] Odionyenma, U. B., Omame, A., Ukanwoke, N. O., & Nometa, I. (2022). Optimal control of Chlamydia model with vaccination. International Journal of Dynamics and Control, 10(1), 332-348.

[6] Akinlotan, M. D., Mallet, D. G., & Araujo, R. P. (2020). An optimal control model of the treatment of chronic Chlamydia trachomatis infection using a combination treatment with antibiotic and tryptophan. Applied Mathematics and Computation, 375, 124899.

[7] Emuoyibofarhe, O. N., Olayiwola, R. O., & Akinwande, N. I. (2015). A Mathematical Model and Simulation of Chlamydia trachomatis in a Human Carrier.

[8] Sharma, S., & Samanta, G. P. (2014). Analysis of a chlamydia epidemic model. Journal of Biological systems, 22(04), 713-744.

[9] Heffernan, C., & Dunningham, J. A. (2009). Simplifying mathematical modelling to test intervention strategies for Chlamydia. Journal of Public Health and Epidemiology, 1(1), 022-030.

[10] Sharomi, O., & Gumel, A. B. (2012). Mathematical study of in-host dynamics of Chlamydia trachomatis. IMA Journal of Applied Mathematics, 77(2), 109-139.

[11] Akinlotan, M. D., Mallet, D. G., & Araujo, R. P. (2020). Mathematical modelling of the role of mucosal vaccine on the within-host dynamics of Chlamydia trachomatis. Journal of Theoretical Biology, 497, 110291.

[12] Atangana, A. (2017). Fractal-fractional differentiation and integration: connecting fractal calculus and fractional calculus to predict complex system. Chaos, solitons & fractals, 102, 396-406.

[13] Farman, M., Hasan, A., Sultan, M., Ahmad, A., Akgl, A., Chaudhry, F., ... & Weera, W. (2023). Yellow virus epidemiological analysis in red chili plants using Mittag-Leffler kernel. Alexandria Engineering Journal, 66, 811-825.

[14] Xu, C., Farman, M., Hasan, A., Akgl, A., Zakarya, M., Albalawi, W., & Park, C. (2022). Lyapunov stability and wave analysis of Covid-19 omicron variant of real data with fractional operator. Alexandria Engineering Journal, 61(12), 11787-11802.

[15] Toufik, M., and Atangana, A. (2017). New numerical approximation of fractional derivative with non-local and non-singular kernel: application to chaotic models. The european physical journal plus, 132, 1-16.

[16] Akgl, E. K. (2019). Solutions of the linear and nonlinear differential equations within the generalized fractional derivatives. Chaos: An Interdisciplinary Journal of Nonlinear Science, 29(2), 023108.

[17] Atangana, A., and Akgl, A. (2020). Can transfer function and Bode diagram be obtained from Sumudu transform. Alexandria Engineering Journal, 59(4), 1971-1984.

[18] Farman, M., Saleem, M. U., Ahmad, A., Imtiaz, S., Tabassum, M. F., Akram, S., and Ahmad, M. O. (2020). A control of glucose level in insulin therapies for the development of artificial pancreas by Atangana Baleanu derivative. Alexandria Engineering Journal, 59(4), 2639-2648.

[19] Sajjad, A., Farman, M., Hasan, A., & Nisar, K. S. (2023). Transmission dynamics of fractional order yellow virus in red chili plants with the CaputoFabrizio operator. Mathematics and Computers in Simulation, 207, 347-368.

[20] Hasan, A., Akgl, A., Farman, M., Chaudhry, F., Sultan, M., and De la Sen, M. (2023). Epidemiological analysis of symmetry in transmission of the Ebola virus with power law kernel. Symmetry, 15(3), 1-29.

[21] Hashemi, M. S. (2018). Some new exact solutions of (2+1)-dimensional nonlinear Heisenberg ferromagnetic spin chain with the conformable time fractional derivative. Optical and Quantum Electronics, 50(2), 1-11.

[22] Aslam, M., Farman, M., Akgl, A., and Sun, M. (2021). Modeling and simulation of fractional order COVID-19 model with quarantined-isolated people. Mathematical Methods in the Applied Sciences, 44(8), 6389-6405.

[23] Farman, M., Ahmad, A., Akgl, A., Saleem, M. U., Naeem, M., and Baleanu, D. (2021). Epidemiological analysis of the coronavirus disease outbreak with random effects. Computers, Materials & Continua, 67(3), 1-13.

[24] Aslam, M., Farman, M., Akgl, A., Ahmad, A., and Sun, M. (2021). Generalized form of fractional order COVID-19 model with MittagLeffler kernel. Mathematical Methods in the Applied Sciences, 44(11), 8598-8614.

[25] Farman, M., Aslam, M., Akgl, A., and Ahmad, A. (2021). Modeling of fractional-order COVID-19 epidemic model with quarantine and social distancing. Mathematical Methods in the Applied Sciences, 44(11), 9334-9350.

[26] Farman, M., Akgl, A., Ahmad, A., Baleanu, D., and Umer Saleem, M. (2021). Dynamical transmission of coronavirus model with analysis and simulation. Computer Modeling in Engineering and Sciences, 127(2), 753-769.

[27] Ivorra, B., Ferrndez, M. R., Vela-Prez, M., and Ramos, A. M. (2020). Mathematical modeling of the spread of the coronavirus disease 2019 (COVID-19) taking into account the undetected infections. The case of China. Communications in nonlinear science and numerical simulation, 88, 105303.

[28] Atangana, A., and Qureshi, S. (2019). Modeling attractors of chaotic dynamical systems with fractalfractional operators. Chaos, solitons and fractals, 123, 320-337.

[29] Nisar, K. S., Farman, M., Hincal, E., Hasan, A., and Abbas, P. (2024). Chlamydia infection with vaccination asymptotic for qualitative and chaotic analysis using the generalized fractal fractional operator. Scientific Reports, 14(1), 1-29.

[30] Rashid, S., Hamidi, S. Z., Raza, M. A., Shafique, R., Alsubaie, A. S., and Elagan, S. K. (2024). Robustness and exploration between the interplay of the nonlinear co-dynamics HIV/AIDS and pneumonia model via fractional differential operators and a probabilistic approach. Scientific Reports, 14(1), 1-42.

[31] Chen, W., Sun, H., Zhang, X., and Koroak, D. (2010). Anomalous diffusion modeling by fractal and fractional derivatives. Computers and Mathematics with Applications, 59(5), 1754-1758.

[32] Atangana, A. (2017). Fractal-fractional differentiation and integration: connecting fractal calculus and fractional calculus to predict complex system. Chaos, solitons and fractals, 102, 396-406.

[33] Ahmad, S., Ullah, A., Al-Mdallal, Q. M., Khan, H., Shah, K., and Khan, A. (2020). Fractional order mathematical modeling of COVID-19 transmission. Chaos, Solitons and Fractals, 139, 110256.

[34] Khan, M. A. (2021). The dynamics of dengue infection through fractal-fractional operator with real statistical data. Alexandria Engineering Journal, 60(1), 321-336.

[35] Vellappandi, M., Kumar, P., & Govindaraj, V. (2023). Role of fractional derivatives in the mathematical modeling of the transmission of Chlamydia in the United States from 1989 to 2019. Nonlinear Dynamics, 111(5), 4915-4929.

[36] Podlubny, I., Chechkin, A., Skovranek, T., Chen, Y., & Jara, B. M. V. (2009). Matrix approach to discrete fractional calculus II: partial fractional differential equations. Journal of Computational Physics, 228(8), 3137-3153.

[37] Samko, S. G. (1993). Fractional integrals and derivatives. Theory and applications.

[38] Caputo, M., & Fabrizio, M. (2015). A new definition of fractional derivative without singular kernel. Progress in Fractional Differentiation & Applications, 1(2), 73-85.

[39] Aydogan, M. S., Baleanu, D., Mousalou, A., & Rezapour, S. (2018). On high order fractional integro-differential equations including the CaputoFabrizio derivative. Boundary Value Problems, 2018(1), 1-15.

[40] Losada, J., & Nieto, J. J. (2015). Properties of a new fractional derivative without singular kernel. Progr. Fract. Differ. Appl, 1(2), 87-92.

[41] Watugala, G. K. (1993). Sumudu transform: a new integral transform to solve differential equations and control engineering problems. Integrated Education, 24(1), 35-43.

[42] Kamocki, R. (2014). Pontryagin maximum principle for fractional ordinary optimal control problems. Mathematical Methods in the Applied Sciences, 37(11), 1668-1686.

[43] Granas, A., & Dugundji, J. (2003). Fixed point theory. New York: Springer, 14, 15-16.