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Fractional Integral Operator Associated with the I-Functions

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Abstract: In this paper we have evaluated certain fractional integral operators associated with Saxenas I-function using the operators of fractional integration. The results obtained are of general character and are reduced to the known results on specializing the parameters. By leveraging the properties of fractional calculus, we investigate new integral transforms involving in this function. We present novel results that extend the existing theory and provide a framework for solving complex integral operators in broadening the scope in this function. These results have potential applications in various fields such as astrophysics, engineering, and applied mathematics, where the I-function and fractional calculus play a crucial role.

Keywords: Fractional Integral operators and I-function.

1. Introduction

Fractional calculus, both in its classical and generalized forms, has played a significant role in developing new classes of special functions. These functions are particularly useful in fractional calculus and integral transforms, as they help simplify complex analytical problems, making them more tractable [1, 2]. One of the fundamental applications of fractional calculus is its ability to define and analyze special functions, including the fractional integral of the H-function with different arguments, which has been extensively studied [3], [4].

In this paper, we explore certain fractional integral and derivative formulas associated with Saxenas I-function, which is defined as follows:

$$I[Z] = I^{m,n}_{p_i,q_i:r}[Z] = \left[Z \left| \begin{matrix} (a_j,A_j)_{1,n}; (a_{ji},A_{ji})_{n+1,p_i} \\ (b_j,B_j)_{1,m}; (b_{ji},B_{ji})_{m+1,q_i} \end{matrix} \right. \right] = \frac{1}{2\pi i}\int_L \Upsilon(s) Z^s ds \tag{1}$$

[5, 6].

where $i = \sqrt{-1}$, and the kernel function $\Upsilon(s)$ is given by:

$$\Upsilon(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - B_j s)\prod_{j=1}^{n}\Gamma(1-a_j+A_j s)}{\sum_{i=1}^{r}(1-b_{ji}+B_{ji}s)\prod_{j=n+1}^{q_i}\Gamma(a_{ji}-A_{ji}s)} \tag{2}$$

[7, 8, 9].

Here, P_i $(i = 1,\dots,r)$ is a finite number, and the parameters A_j, B_j, A_{ji}, B_{ji} are real and positive. The complex parameters a_j, b_j, a_{ji}, b_{ji} must satisfy the condition:

$$a_j(b_h+v) \neq B_h(a_j-1-k), \quad v,k=0,1,2,\ldots,$$
$$h=1,2,\ldots,m; \quad j=1,2,\ldots,r.$$

The contour L extends from $\sigma - i\infty$ to $\sigma + i\infty$ (where σ is real) in the complex s-plane. The contour is chosen such that the points:

$$s=(a_j-1-v),A_j, \quad j=1,2,\ldots,n; \quad v=0,1,2,\ldots$$
$$s=(b_j+v),B_j, \quad j=1,2,\ldots,m; \quad v=0,1,2,\ldots$$

lie to the left-hand side and right-hand side of the contour L, respectively [10, 11].

The subsequent sections of this paper delve into the fractional integral and derivative formulas involving Saxenas I-function, highlighting their significance and applications in various analytical frameworks.

1.1. Fractional Integrals And Derivatives

Let φ,ψ and ζ be complex number, and let $x \in \mathbb{R}_+ = (0,\infty)$. Following Saigo [15][16] fractional integral ($\Re(\varphi)>0$) and ($\Re(\varphi)<0$) first kind of a function $f(x)$ on $\mathbb{R}$ forms:

$$\begin{aligned} I_{0,x}^{\varphi,\psi,\zeta} f &= \frac{x-\varphi-\psi}{\Gamma(\varphi)} \int_0^x (x-t)^{\varphi-1} \quad {}_2F_1\left(\varphi+\psi,-\zeta;\varphi;1-\frac{t}{x}\right) f(t)dt, \quad \Re(\varphi)>0 \\ &= \frac{d_n}{dx^n} I_{0,x}^{\varphi+\psi-\zeta} f, \quad 0<\Re(\varphi)+n\leq 1, \quad (n=1,2,3...) \end{aligned} \tag{3}$$

Where ${}_2F_1(a,b,c)$ is Gauss's hypergeometric function.
Fractional integral ($\Re(\varphi)>0$) and derivative ($\Re(\varphi)<0$) of second kind of a function f(x) on $\mathbb{R}_+$ are given by:

$$\begin{aligned} J_{0,\infty}^{\varphi,\psi,\zeta} f &= \frac{1}{\Gamma(\varphi)} \int_0^\infty (t-x)^{\varphi-\psi} \quad {}_2F_1\left(\varphi+\psi,-\zeta;\varphi;1-\frac{x}{t}\right) f(t)dt, \quad \Re(\varphi)>0 \\ &= (-1)^n \frac{d_n}{dx^n} J_{x,\infty}^{\varphi+n,\psi-n,\zeta} f, \quad 0<\Re(\varphi)+n\leq 1, \quad (n=1,2,3...) \end{aligned} \tag{4}$$

The Riemann-Liouville,Weyl and Erdelyi-Kober fractional operators are interpreted as special cases of the operators I and J as discussed by Saigo [7]

$$\begin{aligned} R_{0,x}^{\varphi} f = I_{0,x}^{\varphi,-\varphi,\zeta} f &= \frac{1}{\Gamma(\varphi)} \int_0^x (x-t)^{\varphi-1} f(t)dt, \Re(\varphi)>0 \\ &= \frac{d_n}{dx^n} R_{0,x}^{\varphi+\zeta} f, 0<\Re(\varphi)+n\leq 1, (n=1,2,3,\ldots) \end{aligned} \tag{5}$$

$$\begin{aligned} W_{0,\infty}^{\varphi} f = J_{0,\infty}^{\varphi,-\varphi,\zeta} f &= \frac{1}{\Gamma(\varphi)} \int_x^\infty (x-t)^{\varphi-1} f(t)dt, \Re(\varphi)>0 \\ &= (-1)^n \frac{d_n}{dx^n} W_{0,\infty}^{\varphi+\zeta} f, 0<\Re(\varphi)+n\leq 1, (n=1,2,3,\ldots) \end{aligned} \tag{6}$$

$$E_{0,\infty}^{\varphi,\zeta} f = I_{0,x}^{\varphi,0,\zeta} f = \frac{1}{\Gamma(\varphi)} x^{-\varphi-\zeta} \int_0^x (x-t)^{\varphi-1} f(t)dt, \Re(\varphi)>0 \tag{7}$$

$$K_{x,\infty}^{\varphi,\zeta} f = J_{x,\infty}^{\varphi,0,\zeta} f = \frac{1}{\Gamma(\varphi)} x^n \int_x^\infty (t-x)^{\varphi-1} t^{-\varphi-\zeta} f(t)dt, \Re(\varphi)>0 \tag{8}$$

The operators I and J defined in (3) and (4) are discussed and applied in many articles. It is notable that the operators can be represented by the Laplace transformation operators and its inverse [16]. In the following discussion we need fractional integral and derivative of a power function studied by Saigo and Rana.

Lemma 1: Let φ, ψ, ζ and $\Im$ be complex numbers. Then there hold the following formulae:

$$I_{0,\infty}^{\varphi,\psi,\zeta} t^{\Im} = \frac{\Gamma(1+Im)\Gamma(1+Im-\psi-\zeta)}{\Gamma(1+Im-\psi)\Gamma(1+Im+\varphi+\zeta)} x^{\Im-\psi} \tag{9}$$

provided that $\Re(\Im) > Max[0, \Re(\psi) - \zeta] - 1$, and

$$I_{0,\infty}^{\varphi,\psi,\zeta} t^{\Im} = \frac{\Gamma(\psi - Im)\Gamma(\zeta - \Im)}{\Gamma(-\Im)\Gamma(\varphi+\psi+\zeta-\Im)} x^{\Im-\psi} \tag{10}$$

If $\Re(\varphi) > 0, \Re(Im) < min[\Re(\psi), \Re(\zeta)], or\ if\ \Re(\varphi) \leq 0, 0 < \Re(\varphi) + n \leq 1$, and $\Re(Im) < min[\Re(\psi), \Re(\zeta)]$, where n is positive integer.

1.2. Fractional Integral And Derivatives Of The I-Function

If φ, ψ, ζ and $\Im$ be complex numbers and in $k > 0$ and

$$|\arg(z)| < \frac{1}{2}\vartheta w, w > 0, \sum_{j=1}^{p} B_j \sum_{j=1}^{p} A_j \leq 0 \tag{11}$$

Then the following result holds:

(I)

$$\begin{aligned} & I_{0,x}^{\varphi,\psi,\zeta} \left\{ t^{\Im} I_{p_i,q_i:r}^{m,n} \left[zt^k \middle| \begin{matrix} (a_j,A_j)_{1,n}, (a_{ji},A_{ji})_{n+1,p_i} \\ (b_j,B_j)_{1,m}, (b_{ji},B_{ji})_{m+1,q_i} \end{matrix} \right] \right\} \\ &= x^{\Im-\psi} I_{p_i+2,q_i+2:r}^{m,n+2} \left[zx^k \middle| \begin{matrix} (-\Im,k), (\psi-\zeta-\Im,k), (a_j,A_j)_{1,n}, (a_{ji},A_{ji})_{n+1,p_i} \\ (b_j,B_j)_{1,m}, (b_{ji},B_{ji})_{m+1,q_i}, (\psi-\Im,k), (-\varphi-\zeta-\Im,k) \end{matrix} \right] \end{aligned} \tag{12}$$

For

$$\Re(\Im) + k\ min_{i \leq j \leq m} \left[\Re\left(\frac{b_j}{B_j} \right) \right] < [-1, \Re(\psi-\zeta) - 1]$$

which can be proved with the help of lemma and explain with figure (1) which is given in below while putting parameters in (12). Similarly, under the same assumptions we have the formula:

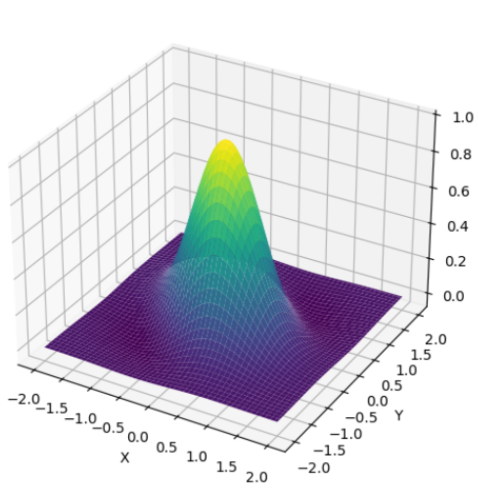

Figure 1: The peak at the centre gradually decreases in all directions.

The image depicts a 3d plot of a function with X and Y as independent variables that creates a bell-shaped curve(likely a Gaussian function.)

(II)

$$J_{0,\infty}^{\varphi,\psi,\zeta}\left\{t^{\Im}I_{p_i,q_i:r}^{m,n}\left[zt^{-k}\left|\begin{matrix}(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i}\end{matrix}\right.\right]\right\}$$
$$=x^{\Im-\psi}I_{p_i+2,q_i+2:r}^{m,n+2}\left[zx^{-k}\left|\begin{matrix}(1-\psi+\Im,k),(1-\zeta-\Im,k),(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i},(1+\Im,k),(1-\varphi+\zeta+\Im,k)\end{matrix}\right.\right] \quad (13)$$

For

$$\Re(\varphi)>0,\Re(\Im)-k\;max_{i\leq j\leq m}\left[\Re\left(\frac{b_j}{B_j}\right)\right]<min[\Re(\psi),\Re(\zeta)]$$

or

$$\Re(\varphi)\leq 0,\quad 0<\Re(\Im)+n\leq 1,\quad \Re(\varphi)-k\;max_{i\leq j\leq m}\left[\Re\left(\frac{b_j}{B_j}\right)\right]<\min\left[\Re(\psi-\zeta),\Re(\zeta)\right]$$

with a positive integer n
which can be proved with the help of lemma and explain with figure (2) which is given in below while putting parameters in (13). Similarly, under the same assumptions we have the formula:

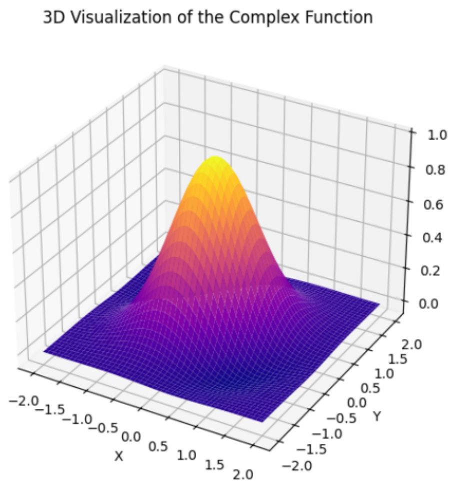

Figure 2: Complex function plotted in real space.

This image show a 3d visualization of a complex function plotted in real space. The graph displays what appears to be the magnitude(abssolute value) of a complex function with two input variable X and Y.

(III)

$$I_{0,x}^{\varphi,\psi,\zeta}\left\{t^{\Im}I_{p_i,q_i:r}^{m,n}\left[zt^{\mu}(x-t)^{\nu}\left|\begin{matrix}(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i}\end{matrix}\right.\right]\right\}$$
$$=\frac{x^{-\psi+\Im}}{\Gamma(\varphi)}\sum_{k=0}^{\infty}\frac{(\varphi+\psi)_k(-\zeta)_k}{(\varphi)_k k!}I_{p_i+2,q_i+1:r}^{m,n+2}\left[zx^{\mu+\nu}\left|\begin{matrix}(-\Im,\mu),(1-\varphi-k,-\nu),(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i},(\varphi-\Im,-k),(-\varphi-\Im-k,\mu+\nu)\end{matrix}\right.\right] \quad (14)$$

provided (in addition to the convergence and existence condition) that

$$\min_{1\leq j\leq m}\left[\Re\left(\frac{b_j}{B_j}\right)\right]>\max\left[\Re\left(\frac{-\Im-1}{\mu}\right),\Re\left(\frac{-\varphi+1}{\nu}\right)\right]$$

which can be proved with the help of lemma and explain with figure (3) which is given in below while putting parameters in (14). Similarly, under the same assumptions we have the formula:

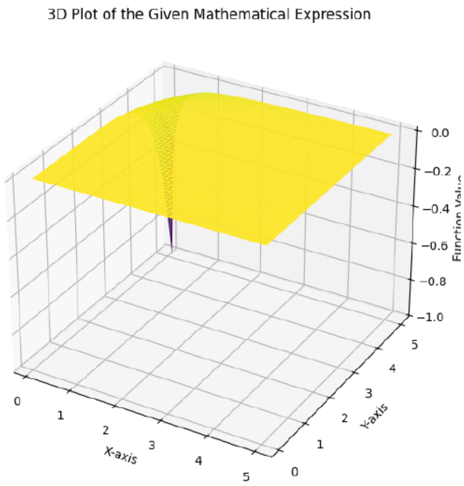

Figure 3: plot The expression with a distinct and unusual feature.

The image show a 3d plot of a mathematical expression with a distinct and unusual feature. This appear to represent a mathematical function with a singularity or pole. *(IV)* setting

$$(h(t,x)) = t^{\Im} I^{m,n}_{p_i,q_i:r}\left[zt^{\mu}(x-t)^{\nu}\middle|\begin{array}{c}(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i}\end{array}\right] \tag{15}$$

We have

$$J^{\varphi,\psi,\zeta}_{0,\infty}\left\{t^{\Im} I^{m,n}_{p_i,q_i:r}\left[zt^{-k}(x-t)^{\nu}\middle|\begin{array}{c}(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i}\end{array}\right]\right\}$$
$$=\frac{x^{-\psi+\Im}}{\Gamma(\varphi)}\sum_{k=0}^{\infty}\frac{(\varphi+\psi)_k(-\zeta)_k}{(\varphi)_k k!}I^{m+1,n+1}_{p_i+2,q_i+1:r}\left[zx^{\mu+\nu}\middle|\begin{array}{c}(1-\psi-k,\mu),(\varphi+\psi-\Im+k,\mu),(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i},(\psi-\Im,\mu+\nu)\end{array}\right] \tag{16}$$

provided (in addition to the convergence and existence condition) that

$$\min_{1\leq j\leq m}\left[\Re\left(\frac{\psi-\Im}{\mu+\nu}\right),\min_{1\leq j\leq m}\left[\Re\left(\frac{b_j}{B_j}\right)\right]\right] > \max\left[\Re\left(\frac{-\varphi}{\nu}\right),\min_{1\leq j\leq m}\left[\Re\left(\frac{a_j-1}{A_j}\right)\right]\right] \tag{17}$$

which can be proved with the help of lemma and explain with figure (4) which is given in below while putting parameters in (16). Similarly, under the same assumptions we have the formula:

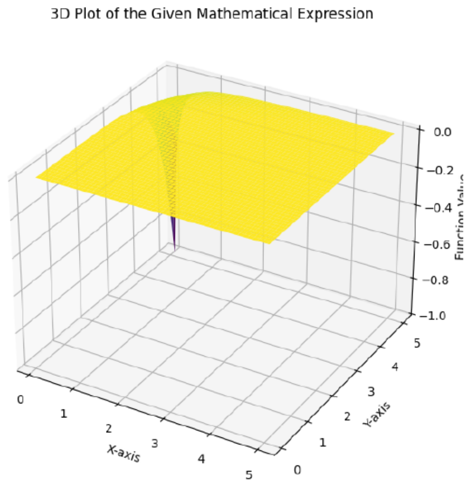

Figure 4: Plot show a mathematical expression with a distinctive feature.

The 3d plot show a mathematical expression with a distinctive feature a mostly flat yellow surface at z=0 with a sharp, narrow spike extending downward at a specific point in the X-Y plane.

Proof: Replacing the I-function by its Mellin-Barnes type integral and interchange the order of integration and summation which permissible under the conditions stated, substituting $t = \frac{x}{4}$ in (4) and then using the beta integral appropriately we get :

$$J^{\varphi,\psi,\zeta}_{0,\infty}\left\{t^{\Im} I^{m,n}_{p_i,q_i:r}\left[zt^{\mu}(x-t)^{\nu}\middle|\begin{array}{c}(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i}\end{array}\right]\right\}$$
$$=\frac{x^{-\psi+\Im}}{\Gamma(\varphi)}\sum_{k=0}^{\infty}\frac{(\varphi+\psi)_k(-\zeta)_k}{(\varphi)_k k!}I^{m+1,n+1}_{p_i+2,q_i+1:r}\left[zx^{\mu+\nu}\middle|\begin{array}{c}(1-\varphi-k,\nu),(\varphi+\psi-\Im+k,\mu),(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i},(\psi-\Im,\mu+\nu)\end{array}\right] \tag{18}$$

which is required result.

Particular Case:

(I) The relation between H and I function is given by

$$I^{m,n}_{p_i,q_i:r}\left[Z\middle|\begin{array}{c}(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i}\end{array}\right]=H^{m,n}_{p_i,q_i:1}\left[Z\middle|\begin{array}{c}(a_1,A_1),(a_2,A_2)...(a_{p_i},A_{p_i})\\(b_1,B_1)(b_2,B_2)(b_{q_i},B_{q_i})\end{array}\right] \tag{19}$$

(II) Putting $\psi = -\varphi$ in (16), it reduces to Weyl fractional integral as

$$W^{\varphi}_{x,\infty}\left\{t^{\Im} I^{m,n}_{p_i,q_i:r}\left[zt^{\mu}(x-t)^{\nu}\middle|\begin{array}{c}(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i}\end{array}\right]\right\}$$
$$=\frac{x^{\varphi+\Im}}{\Gamma(\varphi)}I^{m+1,n+1}_{p_i+2,q_i+1:r}\left[zx^{\mu+\nu}\middle|\begin{array}{c}(1-\varphi,\nu),(-\Im,\mu),(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i},(\varphi-\Im,\mu+\nu)\end{array}\right] \tag{20}$$

(III) Putting ψ in (16) , it reduces to Kober fractional integral, we have

$$K^{\varphi,\zeta}_{x,\infty}\left\{t^{\Im} I^{m,n}_{p_i,q_i:r}\left[zt^{\mu}(x-t)^{\nu}\middle|\begin{array}{c}(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i}\end{array}\right]\right\}$$
$$=\frac{x^{\Im}}{\Gamma(\varphi)}I^{m+1,n+1}_{p_i+2,q_i+1:r}\left[zx^{\mu+\nu}\middle|\begin{array}{c}(1-\varphi,\nu),(\varphi+\zeta-\Im,\mu),(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i},(\zeta-\Im,\mu+\nu)\end{array}\right] \tag{21}$$

If μ is negative, then replacing by $(-\mu)$ and ν is negative then replacing $-\nu$ in (16), we get the following:

$$J_{x,\infty}^{\varphi,\psi,\zeta}\left\{t^{\Im} I_{p_i,q_i:r}^{m,n}\left[zt^{\mu}(x-t)^{-\nu}\left|\begin{array}{c}(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i}\end{array}\right.\right]\right\}$$
$$=\frac{x^{-\psi+\Im}}{\Gamma(\varphi)}\sum_{k=0}^{\infty}\frac{(\varphi+\psi)_k(-\zeta)_k}{(\varphi)_k k!} I_{p_i+2,q_i+1:r}^{m+1,n+1}\left[zx^{-\mu-\nu}\left|\begin{array}{c}(1-\varphi-k,\nu),(\varphi+\psi-\Im+k,\mu),(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,p_i}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,q_i},(\psi-\Im,-\mu-\nu)\end{array}\right.\right] \tag{22}$$

Similarly, we can prove the other results; details are omitted for the sake of brevity.

2. Conclusion

In this study, we have delved into the interplay between fractional integral operators and the I-function is interpretation of H- function unveiling a series of new results the underscore the significance of these mathematical constructs These findings extend the theoretical frameworks and also in new avenues for practical for future to set applications in fields such as mathematical physics, engineering , and applied mathematics. The integration of fractional integral operators with the I-function thus offers a robust tool set for addressing complex problems and advancing research in these domains. To describe memory and heredity effects, this may be used a tool.

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