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Investigation of COVID-19 epidemic mathematical model incorporating media coverage impact and control

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Abstract: In order to estimate and control the recent corona virus disease 2019 outbreak, under media impact. Most countries impose stringent intervention measures to halt the spread of COVID-19. Globally, the COVID-19 epidemic has raised a lot of concerns. A non-smooth SIR system is proposed to investigate the impact of three control strategies (vaccination, therapy, and media coverage) on the spread of an infectious disease. Social media platforms like Face book, Twitter, and others have been crucial in spreading information during the COVID-19 outbreak, both factual and misleading, which has led to widespread confusion. When the susceptible population surpasses the threshold value, the traditional epidemic model comprises media coverage, linear functions that describe vaccine injection, and treatment tactics. COVID-19 model under media coverage investigated both qualitatively and quantitatively. models key proportion are also verified like boundedness, uniqueness and existence. Stability Analysis his also know development on local and global scale in order to properly manage funds and human resources and create successful disease preventive marketing efforts, organizations, institutions, enterprizes, policymakers, and educators who are in charge of infection and disease control must keep a close eye on infection rates.

Keywords: Atangana Toukfik ; Generalized incidence rate; Media coverage;Stability analysis.

1. Introduction

This paper's goal is to examine how awareness coverage and delay affect the management of infectious illnesses. Authors create a SIS model that takes into account how media influences people's behavior. coverage and distinguish between two subcategories of the susceptible class: aware susceptible and unaware susceptible. Media campaigns and infected people are additional model variables. It is believed that media campaigns influence the rate at which people become conscious (unaware), aware to susceptible human, and unaware to susceptible human. Time delays are the intervals of time that pass between an unconscious (awake) sensitive person becoming aware (unaware). The delay needed to set up awareness campaigns are viewed as an additional drawback. The two equilibrium states in the model are the disease-free equilibrium and the endemic equilibrium. In systems with and without delays, the endemic If the fundamental reproduction number is smaller than unity, the disease-free equilibrium is stable, but equilibrium always exhibits a Hopf-bifurcation. The importance of knowledge and delay in reducing the occurrence of infectious diseases is demonstrated by analytical and numerical results. These days, the most dangerous infectious diseases include HIV/AIDS and others. Infectious diseases claim the lives of about 11 million people annually in poor nations, including premature fatalities and deaths of young infants [1,2,3,4]. Though the precise symptoms may differ or depend on the type, most infectious diseases have similar symptoms, including fever, coughing, muscle aches, runny nose, rashes, exhaustion, and diarrhea. Metage nomic sequencing, PCR diagnostics, biochemical testing, microscopy, microbial cultures, and symptomatic diagnostics are the main methods used to identify infectious agents in the diagnosis of infectious diseases [5]. Measles, chickenpox, Ebola, influenza, malaria, dengue, chikungunya, AIDS, TB, hepatitis B, MERS, SARS, and Nipah virus infection are a few of the the common infectious illnesses [6]. Human civilization has been impacted by tuberculosis (TB), a chronic epidemic disease mostly brought on by the bacillus Mycobacterium tuberculosis (Mtb), from prehistoric times. This In 2017, a single infectious agent killed over 1.3 million people and caused 10 million new cases worldwide [7]. The global TB burden is gradually decreasing thanks to improved diagnostic and therapeutic tools. TB mortality decreased by 42 percent overall between 2000 and 2017 [8]. Critical diseases have a number of traits that must be studied and identified in order to reduce or stop their spread [9]. Using the media to inform the public about the disease and the preventive measures that may be taken is the first step in trying to contain an epidemic outbreak. Although media coverage is undoubtedly not the primary element preventing the spread of infectious diseases, it is a crucial issue that requires careful consideration. When there are many infected cases, media coverage may both create panic in the community and, on the one hand, lessen the chance of contact between the vulnerable groups that have been alerted. This aids in limiting the likelihood of transmission and halting the disease's further progress [10]. Globally, there has been a lot of discussion on the role of media in disease prevention, especially in the wake of previous pandemic like COVID-19 and Ebola. The function of the media in spreading It is commonly acknowledged that providing correct information, dispelling myths, and encouraging preventative actions [11]. The proliferation of digital media channels has increased the availability of health-related information globally, impacting public attitudes and actions [12]. Numerous health crises, like HIV/AIDS outbreaks and Ebola, have brought attention to how crucial the media is to preventing disease on the African continent. African Researchers have underlined the importance of situation-specific and culturally sensitive health communication strategies in halting the spread of illness [13]. Additionally, especially in rural and isolated places, the widespread use of mobile technology has enabled creative methods to health message [14]. Furthermore, debates about the moral obligations of journalists and content producers have been sparked by the way social media shapes public perceptions of health issues [15]. Millions of people have died from influenza-related yearly epidemics and sporadic pandemic throughout history. Over the last century, there have been four global pandemic outbreaks of There have been influenza outbreaks in 1918, 1957, 1977, and 2009 [16,17]. About 20,000 Canadians are afflicted by inter-pandemic (or seasonal) influenza, which causes between 2,000 and 8,000 fatalities each year, according to the Public Health Agency of Canada [18]. Flu-related causes may claim the lives of between 3,000 and 49,000 Americans annually, according to reports [19]. During a pandemic or influenza outbreak, the media can have an impact on the spread of disease. The importance of paying attention to health news has grown over the past few decades, so Since media reports are a significant source of information and have the power to influence public behavior, they can be crucial in defining health issues [20]. An important area of research has been examining how variables like vaccinations and incubation times affect the spread of COVID-19 by examining the system's dynamic properties. Abdy and associates [21]. created a fuzzy-parameter COVID-19 S IR Model that takes into consideration a number of factors, including viral load, vaccination, treatment, and following medical advice. Their simulation's results showed that differences Variations in COVID-19 transmission would also be caused by variations in corona virus loads. Likewise, vaccination and following health guidelines had the same effect on slowing or stopping the COVID-19 virus's spread in Indonesia. As stated by Wang et al [22]. The effectiveness of vaccines was based on the amount of time since vaccination in a S VEIR epidemic model that included media impact, age-dependent vaccination, and latency. Evidently, Infectious illness control is greatly aided by vaccinations. A particular vaccine that dramatically lowers the chance of contracting COVID-19 has already been created and garnered a lot of interest. As a reference

[23]. Olorunsaiye et al. discovered that variations in COVID-19 vaccination rates around the world led to varying degrees of COVID-19 immunity in several nations. based on Anderson et al.'s study [24]. Given the pandemic data that is currently accessible, we discovered that, despite a high mortality toll, very few people in Western industrialized countries had received vaccinations. because of the COVID-19 infection, and in order to establish herd immunity, widespread vaccination has been required. As a result, research on vaccine effects in COVID-19 models is required. Zhai and associates [25]. adapted the study of vaccination control in a time-delayed epidemic model to COVID-19. All of these findings highlight the significance of COVID-19 immunization. The emergence of COVID-19 also emphasizes the requirement to promote appropriate vaccination knowledge and raise public awareness of disease prevention. Vaccination rates and public opinion are connected [26, 27]. A fatal infectious disease known as COVID-19, or SARS-CoV-2 (Severe Acute Respiratory Syndrome Corona virus), initially surfaced in Wuhan, China's Hubei province, in the latter part of 2019. Since then, it has dispersed throughout more than 200 nations worldwide, turning into a significant worldwide emergency. Over 5.4 million individuals have been infected and about 0.3 million people have died as a result of the outbreak [28]. The World Health Organization has declared it a global public health emergency (WHO) [29]. One of the WHO's strategies to stop the development of overwhelming diseases is to break the cycle of infection, which includes decreasing human-to-human transmission by lowering secondary infections. infections that affect intimate contacts and health care workers. Thus, factors like social distancing, lock downs, personal hygiene habits, etc., are taken into account. Increasing public knowledge of the disease and its modes of transmission is essential to controlling the outbreak during this crucial phase [30]. One of the best ways to accurately analyze the dynamics of an infectious disease is by mathematical modeling. A few COVID-19 mathematical models have been created and publications that have examined different facets of disease dynamics and potential containment [31,32]. Early on in the pandemic, neither the media nor the general public knew about COVID-19 illnesses. As the disease becomes increasingly well-known, people respond to it and ultimately change their behavior and habits to make themselves less vulnerable [33,34]. People are exposed to the syndromes and possible defense strategies through television and other mass media, including mask wearing, improved cleanliness, social distancing, and the use of preventative ointments. for self-quarantine and protection. People who are aware of the risk of the infection are less likely to get it, which has a big effect on how the pandemic develops [35].
Some definition are recalled from [36].

Definition 1: Atangana Baleanu's partially ordered derivative in Lioville Captuo ABC is provided below and may be expressed as;

$$
{}_{0}^{ABC}D_{t}^{\varpi}\{k(t)\} = \frac{AB(\varpi)}{p-\varpi}\int_{0}^{t}\frac{d^{p}}{dj^{p}}f(j)E_{\varpi}[-\varpi\frac{(t-j)^{\varpi}}{p-\varpi}]dj, p-1<\varpi<p
$$

$AB(0) = AB(1) = 1$ and E_{ϖ} represent the Mittage Lefflar function, respectively. The Laplace transformation may be found here.

$$
[{}_{0}^{ABC}D_{t}^{\varpi}k(t)](f) = \frac{AB(\varpi)}{1-\varpi}\frac{F^{\varpi}L[k(t)](F)-F^{\varpi-1}k(0)}{F^{\varpi}+\frac{g}{g-1}}
$$

Definition 2: With the help of Samudu transformation which is given in equation as follows;

$$
FT\{{}_{0}^{ABC}D_{t}^{\varpi}k(t)\}f = \frac{B(\varpi)}{1-\varpi}(\Pi\varpi(\varpi+1)E_{\varpi}(-\frac{1}{1-\varpi}v^{\varpi}))\times[FT(k(t))-k(0)].
$$

Here A B fractional order integral of ϖ of a function k(t) is given below;

$$
{}_{0}^{ABC}I_{t}^{\varpi}[k(t)] = \frac{1-\varpi}{B-\varpi}k(t)+\frac{\varpi}{B(\varpi)\varpi(\varpi)}\int_{0}^{t}k(f)(t-f)^{\varpi-1}df.
$$

2. COVID-19 System with social media impact

Here, at any moment The number of people susceptible to infection is indicated by $t,S(t)$, those who are now infected with the first disease are represented by $I_1(t)$, and those who are infected with the second are indicated by $I_2(t)$.

illness and the number of those who have recovered from it is represented by $R(t)$. Each of the positive values, C,ι, γ_1, γ_2, μ, ν_1, and ν_2, has a unique biological significance.

Description of Parameters

- C Represent the quantity of infants who enroll in the vulnerable class each time unit.
- ι Represent the population's death rate.
- γ_1 Represent the frequency with which individuals at risk acquire the first illness after coming into contact with those who have already contracted it.
- γ_2 Represent the frequency with which individuals at risk acquire the second disease by coming into contact with those who have already contracted the same ailment.
- ν_1 Represent the rate of recovery from infection among infected individuals who have the initial illness.
- ν_2 Represent the speed at which contagious people who have the second illness recover from their infection.
- μ Represent the pace at which those who have recovered from the first or second illness become less immune.

$$\begin{aligned} \frac{dS}{dt} &= C-\iota S-\gamma_1 SI_1-\gamma_2 SI_2+\mu R, \\ \frac{dI_1}{dt} &= \gamma_1 SI_1-(\iota+\nu_1)I_1, \\ \frac{dI_2}{dt} &= \gamma_2 SI_2-(\iota+\nu_2)I_2, \\ \frac{dR}{dt} &= \nu_1 I_1+\nu_2 I_2-(\iota+\mu)R. \end{aligned} \tag{1}$$

The initial condition
$S(0)=S^0, I_1(0)=I_1^0, I_2(0)=I_2^0, R(0)=R^0$
Using Atangana-Baleanu

$$\begin{aligned} {}^{ABC}D_t^{\chi} S(t) &= C-\iota S-\gamma_1 SI_1-\gamma_2 SI_2+\mu R, \\ {}^{ABC}D_t^{\chi} I_1(t) &= \gamma_1 SI_1-(\iota+\nu_1)I_1, \\ {}^{ABC}D^{\chi} tI_2(t) &= \gamma_2 SI_2-(\iota+\nu_2)I_2, \\ {}^{ABC}D_t^{\chi} R(t) &= \nu_1 I_1+\nu_2 I_2-(\iota+\mu)R, \end{aligned} \tag{2}$$

$$\begin{aligned} {}^{ABC}D_t^{\chi} S(t) &= f(t,V(t)), \\ {}^{ABC}D_t^{\chi} I_1(t) &= g(t,V(t)), \\ {}^{ABC}D_t^{\chi} I_2(t) &= h(t,V(t)), \\ {}^{ABC}D_t^{\chi} R(t) &= i(t,V(t)). \end{aligned}$$

Where $V(t) = S(t), I_1(t), I_2(t), R(t)$, we apply the fractional ${}^{AB}I_t^{\chi}$ of order χ on both sides and utilize Lemma with initial conditions to obtain

$$S(t) = S_0 + \frac{1-\chi}{K(\chi)} f(t, V(t)) + \frac{\chi}{\Gamma(\chi)K(\chi)} \int_0^t (t-x)^{\chi-1} f(x, V(x)) dx,$$

$$I_1(t) = I_1(0) + \frac{1-\chi}{K(\chi)} g(t, V(t)) + \frac{\chi}{\Gamma(\chi)K(\chi)} \int_0^t (t-x)^{\chi-1} g(x, V(x)) dx,$$

$$I_2(t) = I_2(0) + \frac{1-\chi}{K(\chi)} h(t, V(t)) + \frac{\chi}{\Gamma(\chi)K(\chi)} \int_0^t (t-x)^{\chi-1} h(x, V(x)) dx,$$

$$R(t) = R_0 + \frac{1-\chi}{K(\chi)} i(t, V(t)) + \frac{\chi}{\Gamma(\chi)K(\chi)} \int_0^t (t-x)^{\chi-1} i(x, V(x)) dx.$$

Where $V(t) = S(t), I_1(t), I_2(t), R(t)$ and $V(x) = S(x), I_1(x), I_2(x), R(x)$ we have used some conditions to derive uniqueness and existence on linear function f,g.h.i:[0,T]$\times K \times K \longrightarrow K$.

- There exists constants $J_f, J_g, J_h, J_i > 0$ such that for each $S, \bar{S}, I_1, \bar{I_1}, I_2, \bar{I_2}, R, \bar{R} \in R$, such that

$$|f(t, V(t)) - f(\bar{t}, \bar{V(t)})| \leq J_f[|S-\bar{S}| + |I_1 - \bar{I_1}| + |I_2 - \bar{I_2}| + |R-\bar{R}|],$$

$$|g(t, V(t)) - g(\bar{t}, \bar{V(t)})| \leq J_g[|S-\bar{S}| + |I_1 - \bar{I_1}| + |I_2 - \bar{I_2}| + |R-\bar{R}|],$$

$$|h(t, V(t)) - h(\bar{t}, \bar{V(t)})| \leq J_h[|S-\bar{S}| + |I_1 - \bar{I_1}| + |I_2 - \bar{I_2}| + |R-\bar{R}|],$$

$$|i(t, V(t)) - i(\bar{t}, \bar{V(t)})| \leq J_i[|S-\bar{S}| + |I_1 - \bar{I_1}| + |I_2 - \bar{I_2}| + |R-\bar{R}|].$$

where $V(t) = S(t), I_1(t), I_2(t), R(t)$ and $\bar{V(t)} = \bar{S(t)}, \bar{I_1(t)}, \bar{I_2(t)}, \bar{R(t)}$

- There exists constants $A_f, A_g, A_h, A_i, E_f, E_g, E_h, E_i > 0$ and $N_f, N_g, N_h, N_i > 0$

$$|f(t, V(t))| \leq A_f|S| + E_f|S| + N_f,$$

$$|g(t, V(t))| \leq A_g|I_1| + E_g|I_1| + N_g,$$

$$|h(t, V(t))| \leq A_h|I_2| + E_h|I_2| + N_h,$$

$$|i(t, V(t))| \leq A_i|R| + E_i|R| + N_i.$$

where $V(t) = S(t), I_1(t), I_2(t), R(t)$

Theorem 1: Under the continuity of f,g,h,i together with assumption second,system (2) has at least one solution if $(\frac{1-\chi}{K(\chi)})J > 1$, where $J = max\{J_f, J_g, J_h, J_i\}$

Proof: The fixed point theorem of *Krasnoselskii* has the ability to prove existence results. A definition of the operators $F' = (F_1', F_2', F_3', F_4')$ $1G' = (G_1', G_2', G_3', G_4'))$ as indicated below

$$F_1'V_(t) = S_0+\frac{1-\chi}{K(\chi)}f(t,V(t)),$$

$$G_1'V_(t) = \frac{\chi}{\Gamma(\chi)K(\chi)}\int_0^t (t-x)^{\chi-1}f(x,V(x))dx,$$

$$F_2'1(t)V_(t) = I_1(0)+\frac{1-\chi}{K(\chi)}g(t,V(t)),$$

$$G_2'V_(t) = \frac{\chi}{\Gamma(\chi)K(\chi)}\int_0^t (t-x)^{\chi-1}g(x,V(x))dx,$$

$$F_3'V_(t) = I_2(0)+\frac{1-\chi}{K(\chi)}h(t,V(t)),$$

$$G_3'V_(t) = \frac{\chi}{\Gamma(\chi)K(\chi)}\int_0^t (t-x)^{\chi-1}h(x,V(x))dx,$$

$$F_4'V_(t) = R_0+\frac{1-\chi}{K(\chi)}i(t,V(t)),$$

$$G_4'V_(t) = \frac{\chi}{\Gamma(\chi)K(\chi)}\int_0^t (t-x)^{\chi-1}i(x,V(x))dx.$$

Where as,$V(t) = S(t), I_1(t), I_2(t), R(t)$,$Z(x) = S(x), I_1(x), I_2(x), R(x)$ and $V = (S, I_1, I_2, R)$.As a result, F' is a contraction and G' is a continuous operator .For any $V, (\bar{S}, \bar{I_1}, \bar{I_2}, \bar{R}) \in E$, we have;

$$\begin{aligned} |F_1'(t,V(t)) - F_1'(\bar{t},\bar{V(t)})| &\leq \frac{1-\chi}{K(\chi)}J_f[|S-\bar{S}|+|I_1-\bar{I_1}|+|I_2-\bar{I_2}| \\ &+ |R-\bar{R}|]. \end{aligned}$$

Which implies that:

$$\begin{aligned} ||F_1'(t,V(t)) - F_1'(\bar{t},\bar{V(t)})|| &\leq \frac{1-\chi}{K(\chi)}J_f[||S-\bar{S}||+||I_1-\bar{I_1}||+||I_2-\bar{I_2}|| \\ &+ ||R-\bar{R}||], \end{aligned}$$

$$\begin{aligned} ||F_2'(t,V(t)) - F_2'(\bar{t},\bar{V(t)})|| &\leq \frac{1-\chi}{K(\chi)}J_g[||S-\bar{S}||+||I_1-\bar{I_1}||+||I_2-\bar{I_2}|| \\ &+ ||R-\bar{R}||], \end{aligned}$$

$$\begin{aligned} ||F_3'(t,V(t)) - F_3'(\bar{t},\bar{V(t)})|| &\leq \frac{1-\chi}{K(\chi)}J_h[||S-\bar{S}||+||I_1-\bar{I_1}||+||I_2-\bar{I_2}|| \\ &+ ||R-\bar{R}||], \end{aligned}$$

$$
\begin{aligned}
||F_4'(t,V(t)) - F_4'(\bar{t},\bar{V(t)})|| &\leq \frac{1-\chi}{K(\chi)}J_i[||S-\bar{S}|| + ||I_1-\bar{I_1}|| + ||I_2-\bar{I_2}|| \\
&+ ||R-\bar{R}||].
\end{aligned}
$$

Thus:

$$
||F'(t),V(t)) - F'((\bar{t}),\bar{V(t)})|| \leq \frac{1-\chi}{K(\chi)}[||V-(\bar{S},\bar{I_1},\bar{I_2},\bar{R}||].
$$

It demonstrates the contraction nature of F'. The definition of a closed subset B of V is:

$$
E = V \in V : |V|| \leq r, r > 0.
$$

$V = S, I_1, I_2, R$ in this case. The proof that G' is both continuous and compact is as follows: for any $V \in E$, we have:

$$
\begin{aligned}
||G_1'(V)|| \max_{t\varepsilon[0,T]} \frac{\chi}{\Gamma(\chi)K(\chi)} &\int_0^t (t-x)^{\chi-1}f(x,V(x))dx \leq \frac{T^{\chi}}{\Gamma(\chi)K(\chi)}[A_f||S'|| \\
&+ E_f||S|| + N_f],
\end{aligned}
$$

$$
\begin{aligned}
||G_2'(V)|| \max_{t\varepsilon[0,T]} \frac{\chi}{\Gamma(\chi)K(\chi)} &\int_0^t (t-x)^{\chi-1}g(x,V(x))dx \leq \frac{T^{\chi}}{\Gamma(\chi)K(\chi)}[A_g||E'|| \\
&+ E_g||E|| + N_g],
\end{aligned}
$$

$$
\begin{aligned}
||G_3'(V)|| \max_{t\varepsilon[0,T]} \frac{\chi}{\Gamma(\chi)K(\chi)} &\int_0^t (t-x)^{\chi-1}h(x,V(x))dx \leq \frac{T^{\chi}}{\Gamma(\chi)K(\chi)}[A_h||S''|| \\
&+ E_h||D|| + N_h],
\end{aligned}
$$

$$
\begin{aligned}
||G_4'(V)|| \max_{t\varepsilon[0,T]} \frac{\chi}{\Gamma(\chi)K(\chi)} &\int_0^t (t-x)^{\chi-1}i(x,V(x))dx \leq \frac{T^{\chi}}{\Gamma(\chi)K(\chi)}[A_i||E''|| \\
&+ E_i||F|| + N_i].
\end{aligned}
$$

$$
||G'(V)|| \leq T^{\chi}\frac{[(A_1+E_1)s+N_1]}{K(\chi)\Gamma(\chi)} = \Lambda.
$$

Where $N_1 = N_f + N_g + N_h + N_i$, $E_1 = E_f +, E_g + E_h + E_i$, *and* $A_1 = A_f + A_g + A_h + A_i$: Thus, the boundedness of F' is established. Furthermore, we shall demonstrate that F' is *equicontinuous*. Assuming that $t_1 < t_2 \in [0,T]$, we have:

$$
\begin{aligned}
|G_1'(V)(t_2) - G_1'(V)(t_1)| &= \frac{\chi}{\Gamma(\chi)K(\chi)} \times |\int_0^{t_2}(t_2-x)^{\chi-1}f(x,V(x))dx \\
&- \int_0^{t_1}(t_1-x)^{\chi-1}f(x,V(x))dx|, \\
&\leq \frac{\chi}{\Gamma(\chi)K(\chi)}[\int_0^{t_2}(t_2-x)^{\chi-1} \\
&- \int_0^{t_1}(t_1-x)^{\chi-1}]((A_f+E_f)s+N_f)dz, \\
&\leq \frac{((A_f+E_f)s+N_f)}{K(\chi)\Gamma(\chi)}[t_2^{\chi} - t_1^{\chi}],
\end{aligned}
$$

$$|G_2'(V)(t_2) - G_2'(V)(t_1)| \leq \frac{((A_g + E_g)s + N_g)}{K(\chi)\Gamma(\chi)}[t_2^{\chi} - t_1^{\chi}],$$

$$|G_3'(V)(t_2) - G_3'(V)(t_1)| \leq \frac{((A_h + E_h)s + N_h)}{K(\chi)\Gamma(\chi)}[t_2^{\chi} - t_1^{\chi}],$$

$$|G_4'(V)(t_2) - G_4'(V)(t_1)| \leq \frac{((A_i + E_i)s + N_i)}{K(\chi)\Gamma(\chi)}[t_2^{\chi} - t_1^{\chi}].$$

$||G_1'(V)(t_2) - G_1'(V)(t_1)|| \to 0$ as $t_1 \to t_2$ Consequently, $||G'(V)(t_2) - G'(V)(t_1)|| \to 0$ as $t_1 \to t_2$.

Thus, it was demonstrated that G' is an discontinuous operator. Using *Arzel Ascoli*, it has previously been demonstrated that G' is a totally continuous operator and that it is *uniformly* bounded.

Theorem Because of this, G

Theorem 2: If we start with the first assumption, our biomass system with the ABC derivative has a unique solution. $\frac{2T^{\chi}}{K(\chi)\Gamma(\chi)}J < 1$ with $\max_{J_f, J_g, J_h, J_i} = J$

Proof: Define the operator $Q = (Q_1, Q_2, Q_3, Q_4) : V \to V$ as:

$$Q_1V(t) = S_0 + \frac{1-\chi}{K(\chi)} f(t, V(t)) + \frac{\chi}{\Gamma(\chi)K(\chi)} \int_0^t (t-x)^{\chi-1} f(x, V(x))dx,$$

$$Q_2V(t) = E_0 + \frac{1-\chi}{K(\chi)} g(t, V(t)) + \frac{\chi}{\Gamma(\chi)K(\chi)} \int_0^t (t-x)^{\chi-1} g(x, V(x))dx,$$

$$Q_3V(t) = D_0 + \frac{1-\chi}{K(\chi)} h(t, V(t)) + \frac{\chi}{\Gamma(\chi)K(\chi)} \int_0^t (t-x)^{\chi-1} h(x, V(x))dx,$$

$$Q_4V(t) = F_0 + \frac{1-\chi}{K(\chi)} i(t, V(t)) + \frac{\chi}{\Gamma(\chi)K(\chi)} \int_0^t (t-x)^{\chi-1} i(x, V(x))dx.$$

where, $V = (S, I_1, I_2, R)$, $V(t) = (S(t), I_1(t), I_2(t), R(t))$ and $V(x) = (S(x), I_1(t), I_2(t), R(x))$. Now, we take $Z = (S, I_1, I_2, R)$ and $\bar{V} = (\bar{S}, \bar{I_1}, \bar{I_2}, \bar{R})$,we have

$$\begin{aligned} ||Q_1(V) - Q_1(\bar{V})|| &= \max_t \lim[0,T]|| \frac{\chi}{\Gamma(\chi)K(\chi)} \int_0^t (t-x)^{\chi-1} [f(x, V(x))dx \\ &- f(\bar{x}, V\bar{(x)})| \leq \frac{T_{\chi}}{K(\chi)\Gamma(\chi)} J_f [||S - \bar{S}|| + ||I_1 - \bar{I_1}|| \\ &+ ||I_2 - \bar{I_2}|| + ||R - \bar{R}||], \end{aligned}$$

$$\begin{aligned} ||Q_2(V) - Q_2(\bar{V})|| &\leq \frac{T_{\chi}}{K(\chi)\Gamma(\chi)} J_g [||S - \bar{S}|| + ||I_1 - \bar{I_1}|| \\ &+ ||I_2 - \bar{I_2}|| + ||R - \bar{R}||], \end{aligned}$$

$$\begin{aligned} ||Q_3(V) - Q_3(\bar{V})|| &\leq \frac{T_{\chi}}{K(\chi)\Gamma(\chi)} J_h [||S - \bar{S}|| + ||I_1 - \bar{I_1}|| \\ &+ ||I_2 - \bar{I_2}|| + ||R - \bar{R}||], \end{aligned}$$

$$
\begin{aligned}
||Q_4(V)-Q_4(\bar{V})|| &\le \frac{T_\chi}{K(\chi)\Gamma(\chi)}J_i[||S-\bar{S}||+||I_1-\bar{I}_1|| \\
&+ ||I_2-\bar{I}_2||+||R-\bar{R}||].
\end{aligned}
$$

$$
||Q(V)-Q(\bar{V})|| \le \frac{2T_\chi}{K(\chi)\Gamma(\chi)}J||V-\bar{V}||.
$$

Q is hence a contraction. It is in accordance with the Banach contraction theorem that the entire system has a single solution. After that, we present the Ulam-Hyers stability results.

Theorem 3: if the spectral radius of the following matrix is $Ulam-Hyers$ stable, the solution of the investigated model is $Ulam-Hyers$ stable

$\begin{pmatrix} a_1 & a_1 & a_1 & a_1 \\ b_1 & b_1 & b_1 & b_1 \\ c_1 & c_1 & c_1 & c_1 \\ d_1 & d_1 & d_1 & d_1 \end{pmatrix}$ The result of $|a_1+b_1+c_1+d_1|=1$, where

$$
a_1 = (\frac{1-\chi}{K(\chi)}+\frac{T^\chi}{(K(\chi)(\Gamma(\chi)})J_f,
$$

$$
b_1 = (\frac{1-\chi}{K(\chi)}+\frac{T^\chi}{(K(\chi)(\Gamma(\chi)})J_g,
$$

$$
c_1 = (\frac{1-\chi}{K(\chi)}+\frac{T^\chi}{(K(\chi)(\Gamma(\chi)})J_h,
$$

$$
d_1 = (\frac{1-\chi}{K(\chi)}+\frac{T^\chi}{(K(\chi)(\Gamma(\chi)})J_i.
$$

Proof: Consider any model solution, and let $\bar{V}$ represent the model's particular solution, as in:

$$
||V-\bar{V}|| \le \begin{pmatrix} a_1 & a_1 & a_1 & a_1 \\ b_1 & b_1 & b_1 & b_1 \\ c_1 & c_1 & c_1 & c_1 \\ d_1 & d_1 & d_1 & d_1 \end{pmatrix} \begin{pmatrix} ||S-\bar{S}|| \\ ||I_1-\bar{I}_1|| \\ ||I_2-\bar{I}_2|| \\ ||R-\bar{R}|| \end{pmatrix}
$$

Hence, the system is stable.

3. Unique solution of Mathematical Model With Fractional Operator ABC

Theorem 4: In R^4_+, the solution of the specified fractal fractional model, together with beginning conditions, is unique and bounded.

Proof: We possess:

$$
\begin{aligned}
{}^{ABC}D_t^\varsigma(S(t))s &= 0>0, \\
{}^{ABC}D_t^\varsigma(I_1(t))i_1 &= 0>0, \\
{}^{ABC}D_t^\varsigma(I_2(t))i_2 &= 0>0, \\
{}^{ABC}D_t^\varsigma(R(t))r &= 0>0.
\end{aligned}
$$

Equation (3.39), if $(S(0),I_1(0),I_2(0),R(0)) \in R^4_+$, then the solution cannot exit the hyperplane. Furthermore, the vector field for each hyperplane enclosed by the not negative. The domain R^4_+, is a positive invariant set since it is $orthant$.

3.1. Existence and Stability Theory

In this instance, the existence of a minimum single unique solution was explained by the fixed points results. As a result,

$$o^{ABR}D_{0,t}^{\varpi,\iota}(S(t)) = \iota t^{\iota-1}V(K_1),$$

$$o^{ABR}D_{0,t}^{\varpi,\iota}(I_1(t)) = \iota t^{\iota-1}X(K_1),$$

$$o^{ABR}D_{0,t}^{\varpi,\iota}(I_2(t)) = \iota t^{\iota-1}W(K_1),$$

$$o^{ABR}D_{0,t}^{\varpi,\iota}(R(t)) = \iota t^{\iota-1}Q(K_1).$$

where as, $K_1 = (t, S, I_1, I_2, R)$.

$$\begin{aligned} V(K_1) &= C - \iota S - \gamma_1 S I_1 - \gamma_2 S I_2 + \mu R, \\ X(K_1) &= \gamma_1 S I_1 - (\iota + \nu_1) I_1, \\ W(K_1) &= \gamma_2 S I_2 - (\iota + \nu_2) I_2, \\ Q(K_1) &= \nu_1 I_1 + \nu_2 I_2 - (\iota + \mu) R. \end{aligned}$$

We can write above system as:

$$o^{ABR}D_t^{\varpi}\Pi(t) = \iota t^{\iota-1}\Lambda(t, (\Pi(t)).$$

$$\Pi(0) = \Pi_0.$$

By replacing $o^{ABR}D_0^{\varpi,\iota}$ by ${}^{ABC}D_t^{\rho}$ and applying FI, we obtain:

$$\Pi(t) = \Pi(0) + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)}\Lambda(t, \Pi(t)) + \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}.$$

$$\int_0^t \phi^{\iota-1}(t-\phi)^{\iota-1}\Lambda(t, (\Pi(t) d\phi.$$

$$\Pi(t) = S(t),$$

$$\Pi(t) = I_1(t),$$

$$\Pi(t) = I_2(t),$$

$$\Pi(t) = R(t).$$

$$\Pi(0) = S(0),$$

$$\Pi(0) = I_1(0),$$

$$\Pi(0) = I_2(0),$$

$$\Pi(0) = R(0).$$

$$\Lambda(t, (\Pi(t) = V(K_1),$$

$$\Lambda(t, (\Pi(t) = X(K_1),$$

$$\Lambda(t, (\Pi(t) = W(K_1),$$

$$\Lambda(t, (\Pi(t) = Q(K_1).$$

The existence theory uses a Banach space $D = Y \times Y$, where $Y = [0, T]$ as is conventional.

$$||\Pi|| = \max_{t\varepsilon[0,T]} |S(t) + I_1(t) + I_2(t) + R(t)|.$$

Define $\wp : D \to D$ as ;

$$\wp(\Pi)(t) = \Pi(0) + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)}\Lambda(t, (\Pi(t)) + \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}.$$

$$\int_0^t \phi^{\iota-1}(t-\phi)^{\iota-1}\Lambda(t, (\Pi(t)d\phi.$$

Assume that non-linear $\Lambda(t, (\Pi(t)$ meets the above requirements.

- For each $\Pi \in D, \exists$ constants C_Λ and Q_Λ such

$$|\Lambda(t, (\Pi(t) \leq C_\Lambda|\Pi(t)| + Q_\Lambda|.$$

- For each $\Pi, \bar{\Pi} \in D, \exists$ a constant $K_\Delta > 0$ such that:

$$|\Lambda(t, (\Pi(t)) - \Lambda(t, \bar{\Pi(t)})| \leq K_\Lambda|\Pi(t) - \bar{\Pi(t)}|.$$

Theorem 5:

Assume $\Lambda = [0, T] \times D \to O$ be a continuous function. It implies that the model has least single solution.

Proof:

Firstly we will show the operator $\Re$ defined in given definitions 1 and 2 is completely continuous. Since Λ is $continuous.therefore$, $\Re$ is also continuous . Let $J = \{\Pi \in D : ||\Pi|| \leq O, O > 0\}$. Now for any $\Pi \in D, wehave:$

$$\begin{aligned}
|\wp(\Pi)| &= \max_{t\varepsilon[0,T]} |\Pi(0) + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)}\Lambda(t,(\Pi(t)) \\
&+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}\int_0^t \phi^{\iota-1}(t-\phi)^{\iota-1}\Lambda(t,(\Pi(t)d\phi|, \\
&\leq \Pi(0) + \frac{\iota N^{\iota-1}(1-\varpi)}{CB(\varpi)}(C_\Lambda||\Pi|| + Q_\Lambda) \\
&+ \max_{t\varepsilon[0,T]} \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}\int_0^t \phi^{\iota-1}(t-\phi)^{\iota-1}|\Lambda(t,(\Pi(t)|d\phi, \\
&\leq\leq \Pi(0) + \frac{\iota N^{\iota-1}(1-\varpi)}{CB(\varpi)}(C_\Lambda||\Pi|| + Q_\Lambda) \\
&+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}(C_\Lambda||\Pi|| + Q_\Lambda)N^{\varpi+\iota-1}J(\varpi,\iota).
\end{aligned}$$

The $\Re$ is uniformly bounded, but in this case, $J(\varpi,\iota)$ is also continuous. As an illustration of $\Re$ $equicontinuity$, consider $t_1 < t_2 \leq T$. Next, think about

$$\begin{aligned}
|\Re(\Pi)(t_2) - |\Re(\Pi)(t_1)| &= |\frac{\iota t_2^{\iota-1}(1-\varpi)}{CB(\varpi)}\Lambda(t_2,(\Pi(t_2)) \\
&+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}\int_0^{t_2} \phi^{\iota-1}(t_2-\phi)^{\iota-1}\Lambda(\phi,\Pi(\phi))d\phi \\
&- \frac{\iota t_1^{\iota-1}(1-\varpi)}{CB(\varpi)}\Lambda(t_1,(\Pi(t_1)) \\
&+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}\int_0^{t_1} \phi^{\iota-1}(t_1-\phi)^{\iota-1}\Lambda(\phi,\Pi(\phi))d\phi|, \\
&\leq |\frac{\iota t_2^{\iota-1}(1-\varpi)}{CB(\varpi)}(C_\Lambda|\Pi(t)|, Q_\Lambda) \\
&+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}(C_\Lambda|\Pi(t)|, Q_\Lambda)t_2^{\varpi+\iota-1}J(\varpi,\iota) \\
&- |\frac{\iota t_1^{\iota-1}(1-\varpi)}{CB(\varpi)}(C_\Lambda|\Omega(t)|, Q_\Lambda) \\
&+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}(C_\Lambda|\Pi(t)|, Q_\Lambda)t_1^{\varpi+\iota-1}J(\varpi,\iota).
\end{aligned}$$

when $t_1 \to t_2 then|\Re(\Pi)(t_2 - \Re(\Pi)(t_1|$. Consequently we say that $||\Re(\Pi)(t_2 - \Re(\Pi)(t_1|| \to 0, t_1 \to t_2$. Hence, $\Re$ is $discontinuous$. Which completes the proof. **Theorem 6:**

Suppose that if $\rho < 1$. here

$$\rho = (\frac{\iota N^{\iota-1}(1-\varpi)}{CB(\varpi)} + \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}N^{\varpi+\iota-1}J(\varpi,\iota))K_\Lambda.$$

The model in question then has just one solution.

Proof:

Take $\Pi, \bar{\Pi} \in E$, we get

$$|\Re(\Pi) - \Re\bar{\Pi}| = \max_{t\varepsilon[0,T]} |\frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)}(\Lambda(t,\Pi(t)) - \Lambda(t,\bar{\Pi(t)}))$$
$$+\frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}\int_0^t \phi^{\iota-1}(t-\phi)^{\iota-1}d\phi(\Lambda(\phi,\Pi(\phi)) - \Lambda(\phi,\Pi(\bar{\phi})))|,$$
$$\leq [\frac{\iota N^{\iota-1}(1-\varpi)}{CB(\varpi)} + \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)}N^{\varpi+\iota-1}J(\varpi,\iota))]||\Pi - \bar{\Pi}||.$$

Hence ,$\Re$ is a contraction. The suggested model has a one-of-a-kind solution thanks to the the Banach Contraction Principle (BCP) :

3.2. Positiveness and boundness of solutions

Here is a detailed examination of the primaries to show that all of the results make sense because they offer comprehensive explanations of every topic in the globe. This subsection discusses the analysis of different

enables us to be content with the worth of the construction. First, let's talk about class R(t).

$$\begin{aligned} S(t) &= \nu_1 I_1 + \nu_2 I_2 - (\iota+\mu)S, \forall t \geq 0, \\ &\geq -(\iota+\mu)S, \forall t \geq 0, \\ S(t) &\geq S_0 e^{-\iota-\mu}t, \forall t \geq 0. \end{aligned}$$

$$\begin{aligned} R(t) &= \nu_1 I_1 + \nu_2 I_2 - (\iota+\mu)R, \forall t \geq 0, \\ &\geq -(\iota+\mu)R, \forall t \geq 0, \\ R(t) &\geq R_0 e^{-\iota-\mu}t, \forall t \geq 0. \end{aligned}$$

$$\begin{aligned} I_2(t) &= \gamma_2 S I_2 - (\iota+\nu_2)I_2, \forall t \geq 0, \\ &\geq (\iota+\nu_2)I_2, \forall t \geq 0, \\ I_2(t) &\geq I_2(0)e^{\iota+\nu_2}t, \forall t \geq 0. \end{aligned}$$

$$\begin{aligned} I_1(t) &= \gamma_1 S I_1 - (\iota+\nu_1)I_1, \forall t \geq 0, \\ &\geq (\iota+\nu_1)I_1, \forall t \geq 0, \\ I_1(t) &\geq I_1(0)e^{\iota+\nu_1}t, \forall t \geq 0. \end{aligned}$$

We shall define the norm:

$$||h|| = \sup_{t\in K_h} |h(t)|.$$

where K_h denotes $h's$ domain. Using (1.33), for the category $S(t)$, For the function $S(t)$, we get the following inequality.

$$
\begin{aligned}
S(t) &= C-\iota S-\gamma_1 SI_1-\gamma_2 SI_2+\mu R, \forall t \geq 0, \\
S(t) &\geq C+\mu R-(\iota+\gamma_1|I_1|+\gamma_2|I_2|)S, \forall t \geq 0, \\
S(t) &\geq C+\mu R-(\iota+\gamma_1 \sup_{t\in K_h}|I_1|+\gamma_2 \sup_{t\in K_h}|I_2|)S, \forall t \geq 0, \\
S(t) &\geq C+\mu R-(\iota+\gamma_1||I_1||+\gamma_2||I_2||_)S, \forall t \geq 0.
\end{aligned}
$$

This yields:

$$
S(t) \geq e^{C+\mu R-(\iota+\gamma_1||I_1||+\gamma_2||I_2||_)t}, \forall t \geq 0. \tag{3}
$$

3.3. First derivative of Lyapunov

Theorem 7:

The endemic *equilibrium* points R^* of the suggested model are globally asymptotically stable when the reproductive number $R_0 > 1$.

Proof: The $Lyapunov function is denoted by$

using the derivative in relation to t then at this point, their derivative values can be written as follows:

$$
\begin{aligned}
\frac{dL}{dt} &= (\frac{S-S^*}{S})(C-\iota S-\gamma_1 SI_1-\gamma_2 SI_2+\mu R)+(\frac{I_1-I_1^*}{I_1})(\gamma_1 SI_1-(\iota+\nu_1)I_1) \\
&+ (\frac{I_2-I_2^*}{I_2})(\gamma_2 SI_2-(\iota+\nu_2)I_2)+(\frac{R-R^*}{R})(\nu_1 I_1+\nu_2 I_2-(\iota+\mu)R).
\end{aligned}
$$

Putting $S=S-S^*$,$I_1=I_1-I_1^*$,$I_2=I_2-I_2^*$ and $R=R-R^*$ leads to:

$$
\begin{aligned}
\frac{dL}{dt} &= (\frac{S-S^*}{S})(C-\iota(S-S^*)-\gamma_1(S-S^*)(I_1-I_1^*) \\
&- \gamma_2(S-S^*)(I_2-I_2^*)+\mu(R-R^*)) \\
&+ (\frac{I_1-I_1^*}{I_1})(\gamma_1(S-S^*)(I_1-I_1^*)-(\iota+\nu_1)(I_1-I_1^*)) \\
&+ (\frac{I_2-I_2^*}{I_2})(\gamma_2(S-S^*)(I_2-I_2^*)-(\iota+\nu_2)(I_2-I_2^*)) \\
&+ (\frac{R-R^*}{R})(\nu_1(I_1-I_1^*)+\nu_2(I_2-I_2^*)-(\iota+\mu)(R-R^*)).
\end{aligned}
$$

$$
\begin{aligned}
\frac{dL}{dt} &= C\frac{(S-S^*)}{S}-\iota\frac{(S-S^*)^2}{S}-\gamma_1 I_1\frac{(S-S^*)^2}{S}+\gamma_1 I_1^*\frac{(S-S^*)^2}{S} \\
&- \gamma_2 I_2\frac{(S-S^*)^2}{S}+\gamma_2 I_2^*\frac{(S-S^*)^2}{S}+\mu R\frac{(S-S^*)}{S}-\mu R^*\frac{(S-S^*)}{S} \\
&+ \gamma_1 S\frac{(I_1-I_1^*)^2}{I_1}-\gamma_1 S^*\frac{(I_1-I_1^*)^2}{I_1}-\iota\frac{(I_1-I_1^*)^2}{I_1}-\nu_1\frac{(I_1-I_1^*)^2}{I_1} \\
&+ \gamma_2 S\frac{(I_2-I_2^*)^2}{I_2}-\gamma_2 S^*\frac{(I_2-I_2^*)^2}{I_2}-\iota\frac{(I_2-I_2^*)^2}{I_2}-\nu_2\frac{(I_2-I_2^*)^2}{I_2} \\
&+ \nu_1 I_1\frac{(R-R^*)}{R}-\nu_1 I_1^*\frac{(R-R^*)}{R}+\nu_1 I_1\frac{(R-R^*)}{R}-\nu_2 I_2^*\frac{(R-R^*)}{R} \\
&- \iota\frac{(R-R^*)^2}{R}-\mu\frac{(R-R^*)^2}{R}.
\end{aligned}
$$

The *preceding* can be expressed as follows to avoid complications:

$$\frac{dL}{dt} = \Psi_1 - \Psi_2.$$

$$\begin{aligned}\Psi_1 &= C\frac{(S-S^*)}{S} + \gamma_1 I_1^*\frac{(S-S^*)^2}{S} + \gamma_2 I_2^*\frac{(S-S^*)^2}{S} + \mu R\frac{(S-S^*)}{S} \\ &+ \gamma_1 S\frac{(I_1-I_1^*)^2}{I_1} + \gamma_2 S\frac{(I_2-I_2^*)^2}{I_2} + \nu_1 I_1\frac{(R-R^*)}{R} + \nu_1 I_1\frac{(R-R^*)}{R}.\end{aligned}$$

$$\begin{aligned}\Psi_2 &= \iota\frac{(S-S^*)^2}{S} + \gamma_1 I_1\frac{(S-S^*)^2}{S} + \gamma_2 I_2\frac{(S-S^*)^2}{S} + \mu R^*\frac{(S-S^*)}{S} \\ &+ \gamma_1 S^*\frac{(I_1-I_1^*)^2}{I_1} + \iota\frac{(I_1-I_1^*)^2}{I_1} + \nu_1\frac{(I_1-I_1^*)^2}{I_1} + \gamma_2 S^*\frac{(I_2-I_2^*)^2}{I_2} \\ &+ \iota\frac{(I_2-I_2^*)^2}{I_2} + \nu_2\frac{(I_2-I_2^*)^2}{I_2} + \nu_1 I_1^*\frac{(R-R^*)}{R} + \nu_2 I_2^*\frac{(R-R^*)}{R} \\ &+ \iota\frac{(R-R^*)^2}{R} + \mu\frac{(R-R^*)^2}{R}.\end{aligned}$$

It is concluded that if $\Psi_1 < \Psi_2$, this yields $\dot{L} < 0$, however when $S = S^*, I_1 = I_1^*, I_2 = I_2^*$, and $R = R^*$

$$0 = \Psi_1 - \Psi_2.$$

$$\frac{dL}{dt} = 0.$$

It is evident that the proposed model contains the largest set of compact invariants as.

$$(S^*, I_1^*, I_2^*, R^*) \in \Gamma; \frac{dL}{dt} = 0.$$

is what's important. The endemic equilibrium of the model under consideration is S_*. S_* is globally asymptotically stable in $\Gamma if \Psi_1 - \Psi_2$, according to the *Lasalles* invariance idea.

4. Numerical Algorithm with Proposed Scheme

The numerical techniques for the suggested models are now created in the section that follows using the ABC fractal operator and $La-grangian$ piece wise *interpolation*. The entire set of data is based on the Adams-Bashforth (AB) technique. By now,

Extending the system in the Caputo sense and employing the integral in the ABC sense:

$$S(t) = S(0) + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_1(K_1) \quad + \quad \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \int_0^t \Psi J_1(K_2) d\chi,$$

$$I_1(t) = I_1(0) + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_2(K_1) \quad + \quad \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \int_0^t \Psi J_2(K_2) d\chi,$$

$$I_2(t) = I_2(0) + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_3(K_1) \quad + \quad \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \int_0^t \Psi J_3(K_2) d\chi,$$

$$R(t) = R(0) + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_4(K_1) \quad + \quad \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \int_0^t \Psi J_4(K_2) d\chi.$$

where $K_1 = (t, S, I_1, I_2, R), \chi^{\iota-1}(t-\chi)$ and $K_2 = (\chi, S, I_1, I_2, R)$. Now , at $t = t_y + 1$, we get:

$$\begin{aligned} S^{y+1} &= S^0 + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_1(t_y, S^y, I_1^y, I_2^y, R^y) \\ &+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \int_0^t \chi^{\iota-1}(t-\chi)^{\varpi-1} J_1(\chi, S, I_1, I_2, R) d\chi, \end{aligned}$$

$$\begin{aligned} I_1^{y+1} &= I_1^0 + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_2(t_y, S^y, I_1^y, I_2^y, R^y) \\ &+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \int_0^t \chi^{\iota-1}(t-\chi)^{\varpi-1} J_2(\chi, S, I_1, I_2, R) d\chi, \end{aligned}$$

$$\begin{aligned} I_2^{y+1} &= I_2^0 + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_3(t_y, S^y, I_1^y, I_2^y, R^y) \\ &+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \int_0^t \chi^{\iota-1}(t-\chi)^{\varpi-1} J_3(\chi, S, I_1, I_2, R) d\chi, \end{aligned}$$

$$\begin{aligned} R^{y+1} &= R^0 + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_4(t_y, S^y, I_1^y, I_2^y, R^y) \\ &+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \int_0^t \chi^{\iota-1}(t-\chi)^{\varpi-1} J_4(\chi, S, I_1, I_2, R) d\chi. \end{aligned}$$

On *RHS* we have used the approximation of integrals and hence we obtained:

$$\begin{aligned} S^{y+1} &= S^0 + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_1(t_y, S^y, I_1^y, I_2^y, R^y) \\ &+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \sum_{f=0}^{y} \int_0^t \chi^{\iota-1}(t-\chi)^{\varpi-1} J_1(\chi, S, I_1, I_2, R) d\chi, \end{aligned}$$

$$\begin{aligned} I_1^{y+1} &= I_1^0 + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_2(t_y, S^y, I_1^y, I_2^y, R^y) \\ &+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \sum_{f=0}^{y} \int_0^t \chi^{\iota-1}(t-\chi)^{\varpi-1} J_2(\chi, S, I_1, I_2, R) d\chi, \end{aligned}$$

$$\begin{aligned} I_2^{y+1} &= I_2^0 + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_3(t_y, S^y, I_1^y, I_2^y, R^y) \\ &+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \sum_{f=0}^{y} \int_0^t \chi^{\iota-1}(t-\chi)^{\varpi-1} J_3(\chi, S, I_1, I_2, R) d\chi, \end{aligned}$$

$$
\begin{aligned}
R^{y+1} &= R^0 + \frac{\iota t^{\iota-1}(1-\varpi)}{CB(\varpi)} J_4(t_y, S^y, I_1^y, I_2^y, R^y) \\
&+ \frac{\varpi\iota}{CB(\varpi)\Gamma(\varpi)} \sum_{f=0}^{y} \int_0^t \chi^{\iota-1}(t-\chi)^{\varpi-1} J_4(\chi, S, I_1, I_2, R) d\chi.
\end{aligned}
$$

$$
\begin{aligned}
L_f(\chi) &= \frac{\chi - t_f - 1}{t_f - t_f - 1} t_f^{\iota-1} J_1(t^f, S^f, I_1^f, I_2^f, R^f) \\
&- \frac{\chi - t_f}{t_f - t_f - 1} t_{f-1}^{\iota-1} J_1(t^{f-1}, S^{f-1}, I_1^{f-1}, I_2^{f-1}, R^{f-1}),
\end{aligned}
$$

$$
\begin{aligned}
O_f(\chi) &= \frac{\chi - t_f - 1}{t_f - t_f - 1} t_f^{\iota-1} J_2(t^f, S^f, I_1^f, I_2^f, R^f) \\
&- \frac{\chi - t_f}{t_f - t_f - 1} t_{f-1}^{\iota-1} J_2(t^{f-1}, S^{f-1}, I_1^{f-1}, I_2^{f-1}, R^{f-1}),
\end{aligned}
$$

$$
\begin{aligned}
P_f(\chi) &= \frac{\chi - t_f - 1}{t_f - t_f - 1} t_f^{\iota-1} J_3(t^f, S^f, I_1^f, I_2^f, R^f) \\
&- \frac{\chi - t_f}{t_f - t_f - 1} t_{f-1}^{\iota-1} J_3(t^{f-1}, S^{f-1}, I_1^{f-1}, I_2^{f-1}, R^{f-1}),
\end{aligned}
$$

$$
\begin{aligned}
U_f(\chi) &= \frac{\chi - t_f - 1}{t_f - t_f - 1} t_f^{\iota-1} J_4(t^f, S^f, I_1^f, I_2^f, R^f) \\
&- \frac{\chi - t_f}{t_f - t_f - 1} t_{f-1}^{\iota-1} J_4(t^{f-1}, S^{f-1}, I_1^{f-1}, I_2^{f-1}, R^{f-1}).
\end{aligned}
$$

$$
\begin{aligned}
S^{y+1} &= S^0 + \frac{\iota t_y^{\iota-1}(1-\varpi)}{CB(\varpi)} J_1(t_y, S^y, I_1^y, I_2^y, R^y) \\
&+ \frac{\iota(\Theta t)^{\varpi}}{CB(\varpi)\Gamma(\varpi+2)} \sum_{M}^{y} = 0[t_f^{\iota-1} J_1(t^f, S^f, I_1^f, I_2^f, R^f) \\
&\times \Lambda - t_{f-1}^{\iota-1} J_1(t_{f-1}, S^{f-1}, I_1^{f-1}, I_2^{f-1}, R^{f-1}) \times ((1+y-f)^{\varpi+1} \\
&- (y-f)^{\varpi}(1+\varpi+y-f))],
\end{aligned}
$$

$$
\begin{aligned}
I_1^{y+1} &= I_1^0 + \frac{\iota t_y^{\iota-1}(1-\varpi)}{CB(\varpi)} J_2(t_y, S^y, I_1^y, I_2^y, R^y) \\
&+ \frac{\iota(\Theta t)^{\varpi}}{CB(\varpi)\Gamma(\varpi+2)} \sum_{M}^{y} = 0[t_f^{\iota-1} J_2(t^f, S^f, I_1^f, I_2^f, R^f) \\
&\times \Lambda - t_{f-1}^{\iota-1} J_2(t_{f-1}, S^{f-1}, I_1^{f-1}, I_2^{f-1}, R^{f-1}) \times ((1+y-f)^{\varpi+1} \\
&- (y-f)^{\varpi}(1+\varpi+y-f))],
\end{aligned}
$$

$$
\begin{aligned}
I_1^{y+1} &= I_1^0+\frac{\iota t_y^{\iota-1}(1-\varpi)}{CB(\varpi)}J_3(t_y,S^y,I_1^y,I_2^y,R^y)\\
&+ \frac{\iota(\Theta t)^{\varpi}}{CB(\varpi)\Gamma(\varpi+2)}\sum_{M}^{y}=0[t_f^{\iota-1}J_3(t^f,S^f,I_1^f,I_2^f,R^f)\\
&\times \Lambda-t_{f-1}^{\iota-1}J_3(t_{f-1},S^{f-1},I_1^{f-1},I_2^{f-1},R^{f-1})\times((1+y-f)^{\varpi+1}\\
&- (y-f)^{\varpi}(1+\varpi+y-f))],
\end{aligned}
$$

$$
\begin{aligned}
R^{y+1} &= R^0+\frac{\iota t_y^{\iota-1}(1-\varpi)}{CB(\varpi)}J_4(t_y,S^y,I_1^y,I_2^y,R^y)\\
&+ \frac{\iota(\Theta t)^{\varpi}}{CB(\varpi)\Gamma(\varpi+2)}\sum_{M}^{y}=0[t_f^{\iota-1}J_4(t^f,S^f,I_1^f,I_2^f,R^f)\\
&\times \Lambda-t_{f-1}^{\iota-1}J_4(t_{f-1},S^{f-1},I_1^{f-1},I_2^{f-1},R^{f-1})\times((1+y-f)^{\varpi+1}\\
&- (y-f)^{\varpi}(1+\varpi+y-f))].
\end{aligned}
$$

where $((y+1-f)^{\varpi}(y-f+2+\varpi)-(y-2)\varpi(2+2\varpi+y-2))$ reflects the considered model's overall numerical findings .

5. Mathematical Analysis by ABC technique

Approximation system solution is achieved using the Sumudu transformer operator. The following is how the operator is applied to the both ends of the proposed system:

$$\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{1-\alpha}E_\alpha(-\frac{1}{1-\alpha}w^\alpha)ST[S(t)-S(0)]=ST[\lambda+\chi R-(\xi+\mu)S],$$

$$\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{1-\alpha}E_\alpha(-\frac{1}{1-\alpha}w^\alpha)ST[I(t)-I(0)]=ST[\xi S-(\mu+\gamma)I],$$

$$\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{1-\alpha}E_\alpha(-\frac{1}{1-\alpha}w^\alpha)ST[T(t)-T(0)]=ST[p\gamma I-(\mu+\sigma)T],$$

$$\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{1-\alpha}E_\alpha(-\frac{1}{1-\alpha}w^\alpha)ST[R(t)-R(0)]=ST[(1-p)\gamma I+\sigma T-(\mu+\chi)R].$$

6. The physical explanations

We employed the Atangana-Toufik Technique (ABC) of the Covid-19 model with media effects under preset initial conditions to investigate disease transmission employing simulations that comprised both asymptomatic and vaccination modes of transfer. We may utilize fractional values to find a nonlinear system. The simulation data for all sub-compartments are displayed in Figure 3-1 to figure 3-4 with different fractional order values. The effectiveness of the obtained theoretical implications is illustrated by a number of cases. Reliable findings are obtained when non-integers parametric possibilities are used for Covid-19 disease, taking consideration of both symptomatic and asymptomatic propagation.

The system's initial conditions are as follows: $S_0(t) = S(0), I_0(t) = I(0), T_0(t) = T(0), R_0(t) = R(0)$ and parameters values are $C = 0.08, \iota = 0.011, \gamma_1 = 0.8, \gamma_2 = 0.0076, \mu = 0.6, \nu_1 = 0.21$ and $\nu_2 = 0.11$. Reliable and accurate findings are produced for each compartment at derivatives of non-integer order by the investigation using an advanced numerical approach. Reducing fractional values increases the credibility of the findings.

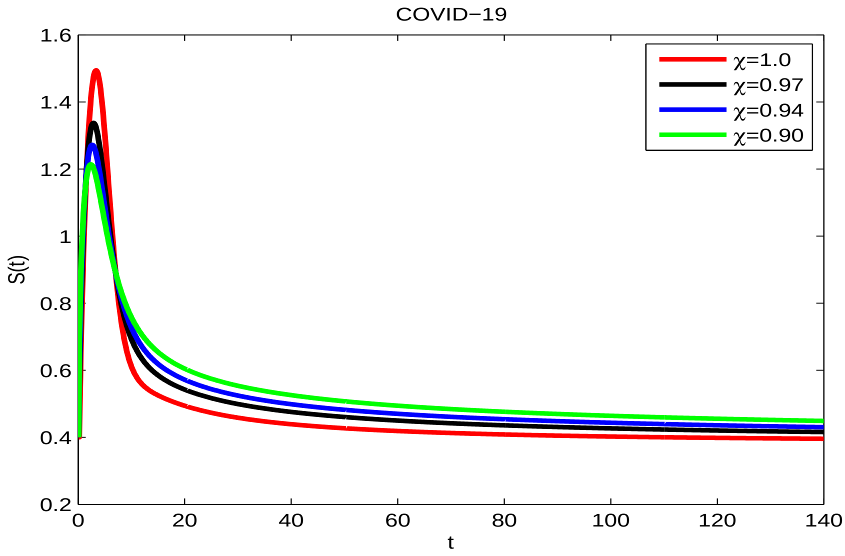

Figure 1: $S(t)$, Simulation for developed solution.

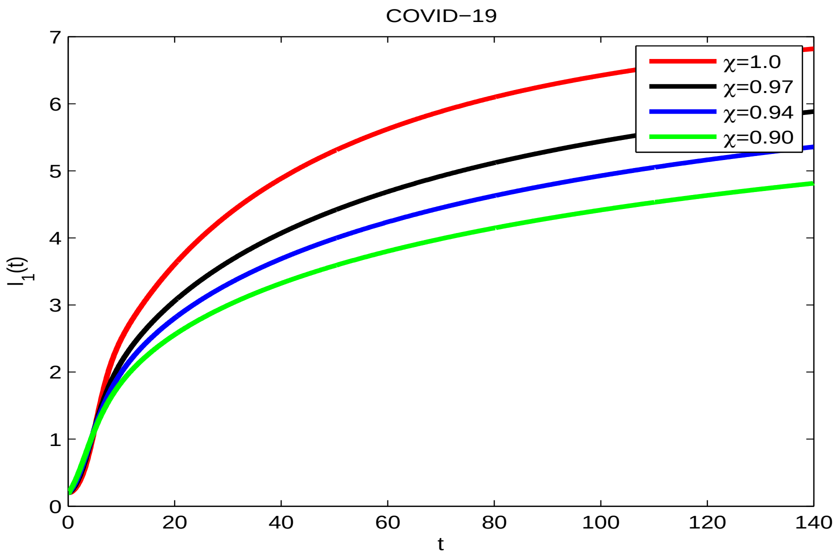

Figure 2: $I_1(t)$, Simulation for developed solution.

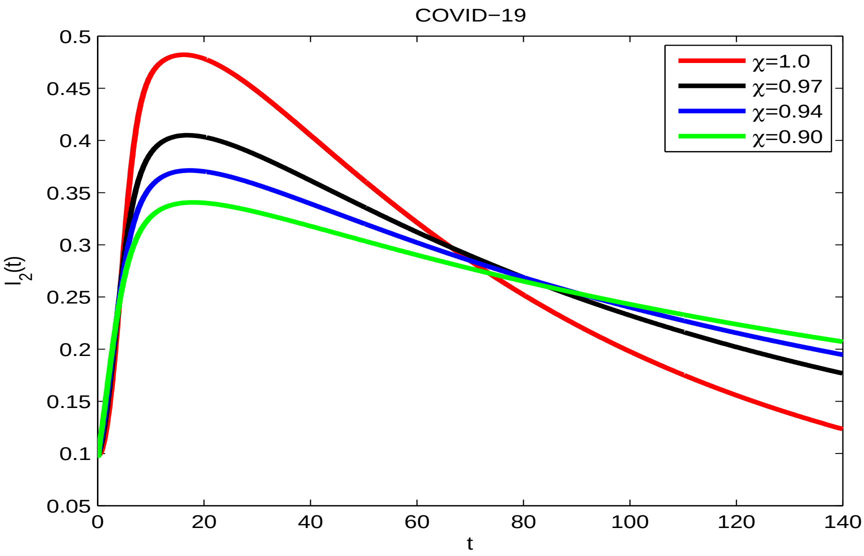

Figure 3: $I_2(t)$, Simulation for developed solution.

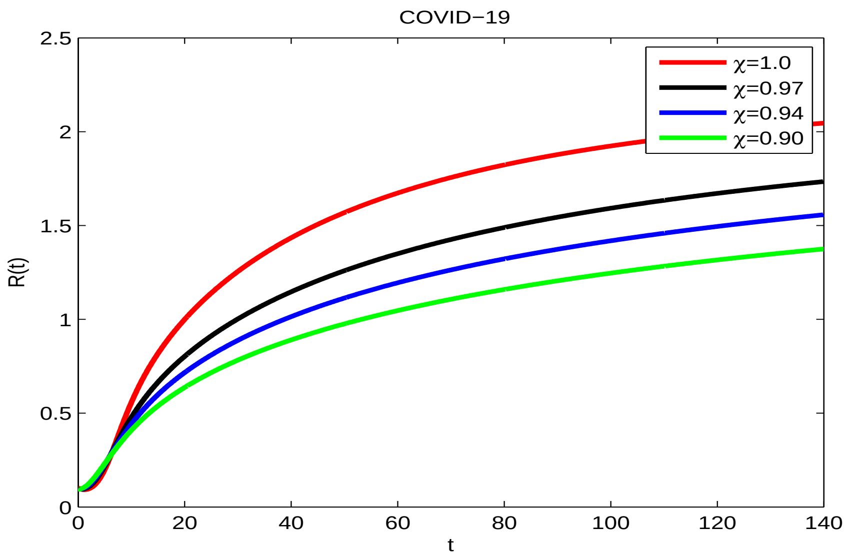

Figure 4: $R(t)$, Simulation for developed solution.

7. Conclusion

We investigate the effects of vaccination and media attention on COVID-19 transmission based on the available data. This study's primary goal is to investigate the combined effects of media coverage and vaccination on COVID-19 prevalence. This work has constructed a general model on the dynamics of infectious diseases, taking media awareness into account as a control variable function. Mathematical model is analysis for unique, and bounded solution. Also verify the existence of solution including stability Analysis. Advance Approach is utilized to find the solution of the COVID-19 model with media effects including vaccination and verify it by simulation using codding on MATLAB. It has been developed that the despite some tiredness, the news and data gathered by the media consistently show a beneficial influence that serves as a pre-recovery alternative for reducing the spread of the virus. This is especially important during times when there are few effective vaccines or appropriate treatments available. Second, while media coverage has the power to alter people's behavior, our research shows this in a very basic way the rate function of mask wearing reflects the impact of media education on the spread of COVID-19. In general, our research shows that media publicity and immunization significantly reduce the spread of COVID-19. The better way of media coverage will impact to control the disease, It will also helpful to develop future control strategies as well as awareness for control purpose.

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